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Policy Coordination and Convergence in the EU

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Tilburg University

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Proefschrift

ter verkrijging van de graad van doctor aan de Katholieke Universiteit Brabant, op gezag van de rector magnificus, prof. dr. L.F.W. de Klerk, in het openbaar te verdedigen ten overstaan van een door het college van dekanen aangewezen commissie in de aula van de Universiteit op

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Rudy Clemens Douven

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Contents

Chapter 1

Introduction

These days there is a lot of debate about the creation of an Economic and Monetary Union (EMU) and the transition in stages towards it. Besides the economic aspects one should, for sure, not neglect the cultural, social and political aspects of this ongoing EMU process in the European Union (EU). Taking all the consequences of the different aspects of this integration process into consideration and weighing out the pros and cons is extremely difficult, if not impossible. Also the opinions on how and how far the EMU process should proceed vary not only by country but also through time. A nice example of this last aspect is Denmark where the Danish people rejected the Maastricht Treaty in a referendum on June 2, 1992. However, almost one year later the Danish people approved the Maastricht Treaty in a referendum on May 18, 1993.

This thesis attempts to enlighten some economic aspects of the EMU process. In particular, we study the role of the European Commission in this process from a global macroeconomic perspective. The analyses will be carried out in an international policy coordination framework and will focus on the implications of the growing international macroeconomic interdependence between various European economies.

1.1 The European Commission

In the literature the European Commission (EC) is described as a supranational institution whose broad objective is currently to guide the EU-Member States in their transition towards an Economic and Monetary Union (EMU) (see Molle [56]). Most of these objectives are elaborated in EC treaties. Such a treaty must be seen as a rough framework: specific rules and regulations still have to be worked out by various institutions of the EC. The most important institutions are the Commission and the Council of Ministers. They are jointly responsible for coordinating national policies, issuing regulations and directives and taking decisions in areas not foreseen in the treaties. Further they take care of the supervision of the treaties. Important in the decision process of the EC is that regulations, directives, decisions and recommendations are initially born from a Community's point of view. Further, the specific structure of the EC can be called supranational, which means that created regulations and laws by the EC institutions overrule national regulations and laws. The most important source of income of the EC are the contributions by each Member State, which yearly pay a certain percentage of their GDP to the Commission. Another important source of income are the value added tax (VAT) income in every country. A fixed percentage of the VAT received by each Member State has to be transferred to the EC. At this moment the total EC budget is about 1.5% of the aggregate of the Member States. This is a relatively small figure and since it has to be transferred back to the Member States (according to Article 199 of the EC Treaty) via a list of expenditure programmes in different fields, the EC can only exert no more than a little direct influence on the total economy. So in conclusion one can say that there certainly will be some redistribution effects but for an effective macro-economic policy the total EC budget must be considered too small.

1.2 The EMU process

The process of the creation of an Economic and Monetary Union started with the Werner Report which was completed in 1970 (see Steinherr [72]). This Report describes a transition towards a monetary union in various stages and can be seen as a predecessor of the Delors Report which was published in 1989 (see Delors [18]). Both Reports report a gradual approach towards EMU. However, there were some differences in the final interpretation of EMU and the transition in stages towards it. One of the differences is the role set out for the European Commission. In the Werner Report the Commission should envisage more central control of national fiscal policies than in the Delors Report. The Werner Report also sketches the European Commission as the centre of decision for economic policy which will exercise independently, in accordance with the Community interest, a decisive influence over the general economic policy of the Community. However, in a concession to political realities, the Delors report did not propose transferring control of national budgetary policies to the Community but instead simply recommended that the Community should 'monitor the overall economic situation' and should 'assess the consistency of developments in individual countries with regard to common objectives' (see also Eichengreen [24]).

As a follow up of the Delors report, the Treaty designed at Maastricht 1991 was a major step forward in the EMU process. The Maastricht Treaty describes the realization of a monetary union in three stages. The Treaty also maps out conditions and a timetable for countries in order to proceed to Stage III of EMU which is characterised by the establishment of an independent European central bank and transferring to it the responsibility for the conduct of monetary policy. The four conditions specified at the Maastricht Treaty for admitting an EU-country to an EMU are specified as follows:
(i) A consumer price inflation of no more than 1.5 percentage points above the average of the three countries with lowest inflation rates.
(ii) An average nominal long term interest rate of no more than 2 percentage points above the average for the three EU-countries with lowest inflation rates.
(iii) No exchange rate realignments for at least two years.
(iv) A sustainable government financial position, defined as a general government deficit to GDP ratio of less than 3 percent and a gross government debt of less than 60 percent of GDP.
The purpose of these four rules are clear, it is to prevent the EMU being destabilised by the premature admission of an EU-country whose fundamentals are not yet compatible with a fixed exchange rate (see Bean [4]).

In order to guide this process of closer convergence and increasing integration the European Commission sets out broad policy guidelines for the EU-Member States (see, e.g., the 1994 Broad Economic Policy Guidelines as published by the European Commission [16]). These broad guidelines for economic policy provide targets for economic policy in the short and medium term and are specified such that if EU-countries follow these guidelines they will be able to participate at Stage three of EMU, which has been provisionally dated at January 1st, 1999.

1.3 International policy coordination

As the Delors Report states [18, page 5]:'By greatly strengthening economic interdependence between member countries, the single market will reduce the room for independent policy manoeuvre and amplify the cross-border effects of developments originating in each member country', and on page 6:'The integration process thus requires more intensive and effective policy coordination, even within the framework of the present exchange rate arrangements, not only in the monetary field but also in areas of national economic management affecting aggregate demand, prices and costs of production'. The Delors Report recommends a stronger tightening of international policy coordination among the EU-countries which should be guided by the European Commission. Thus the success of the internal market hinges on the one hand to a decisive extent on a much closer coordination of national economies as well as on the other hand on more effective Community policies. The Delors Report reports however a warning note since the decision-making of national governments may fall between two stools. This is nicely stated at page 6 of the Report: 'Decision-making authorities are subject to many pressures and institutional constraints and even the best efforts to take into account the international repercussions of their policies are likely to fail at certain times. While voluntary co-operation should be relied upon as much as possible to arrive at increasingly consistent national policies, thus taking account of divergent constitutional situations in member countries, there is also likely to be a need for more binding procedures.' It is precisely this tension between the objectives of national policy-makers and the international binding procedures set out by the Commission to which a main part of this thesis is devoted. In this thesis we use policy coordination (and also cooperation) in the sense that it refers to decision making that maximises joint welfare and thereby tries to exploit international interdependencies positively. Maximising joint welfare is done by jointly setting the national instruments. On the other hand a lack of coordination means that each country sets its own instruments and tries to maximise its own welfare. In a game theoretic context the latter situation can be viewed as a non-cooperative outcome which corresponds to a Nash equilibrium whereas the first situation can be viewed as a Pareto efficient outcome. For an explanation of the various terms in this field, which are often used in different ways with different meanings, we refer to Horne and Masson [46]. The policy coordination literature is growing fast and to give an extensive overview would already be a severe task. We therefore restrict ourselves and mention only briefly some main contributions in this field. In the next subsections we, firstly, very briefly mention some important contributions to the policy coordination literature and, secondly, we will discuss a side path of this literature called hierarchical control.

1.3.1 The international policy coordination literature

The main impulse to the empirical part of the literature probably originates from the paper by Oudiz and Sachs [61] published in 1984. They provided the first empirical estimates of the gains of coordination, which they found to be rather small. Subsequently there was a lot of research about the gains (and also possible losses) of international policy coordination. An example is Holtham and Hughes Hallett [45]. In that research the robustness of estimates of policy coordination gains and non-cooperative losses were investigated with ten different macro-economic models. All the experiments predicted rather large cooperation gains but they found a considerable variation in the optimal strategies. This observation

raised the question in the literature whether discretionary policy rules are really preferred over simple policy rules. This lead to a branch of research which investigated whether simple policy rules could emulate full policy coordination reasonably well. For a more extensive overview on this line of the literature we refer to Gosh and Masson [34] and McKibbin and Sachs [55] and their references.
A related line of research is the study of dynamic aspects of policy-making. The main research in economics on this subject probably started with the controversial article of Kydland an Prescott [54]. A lot of ideas introduced in this branch of literature found its origin in game theory and control literature. A main work in the game theoretical field is Başar and Olsder [3], which is often quoted by economists. This work presents also a good mathematical description of the various concepts used in (dynamic) game theory, such as information patterns and (strong) time (in)consistency problems. A lively economic discussion about various types of non-cooperative game outcomes, information patterns and time (in)consistency problems can be found in de Zeeuw and van der Ploeg [86] and the comments on that paper by Hughes Hallett [36]. A thorough description of the use of optimal control in macro-economic models, which also discusses related economic problems such as uncertainty, the Lucas critique, rational expectations and the (re)construction of preferences can be found in Brandsma [8] and Petit [66].
Another line of important research deals with uncertainty. In the empirical policy coordination uncertainty glooms everywhere. For a recent good overview of this literature we refer to Ghosh and Masson [34].
A thorough description of the mathematical formulars for calculating various types of game outcomes for economic problems specified in a linear quadratic framework can be found in de Zeeuw [85].
A recent collection of articles about the relevance and reality of international policy coordination is Blommestein [7] which contains also a contribution of Frenkel, Goldstein and Masson [32] with a survey of key issues in the international coordination of economic policies. We refer also to the references of this article for a more extensive overview of the literature.

For the remainder of this thesis we ignore the issue of uncertainty and consider only a deterministic framework. Furthermore, we assume that countries who participate at the game have complete information. Thus we focus on the application of dynamic game theory and measure outcomes under cooperative and non-cooperative agreements in one 'true' model. This is already a severe assumption but it appears that even if we would know the 'true' model, then game aspects provide such a richness of complexity that good future predictions are still almost unattainable.

1.3.2 Hierarchical control

A less developed part in the economic policy coordination literature is the one on hierarchical control. The introduction of hierarchical control models in economics probably originates from researchers of the former Eastern Europe. At that time it was popular among these researchers to model their plan-economy as a certain type of hierarchical control system. Simply stated, a hierarchical control system can be viewed as a system where the coordinator's policy strategies at the upper level are transmitted (and often imposed) to the subsystems at the lower levels. In the literature hierarchical control is often compared with outcomes using decentralised control (see, e.g., Jamshidi [50] and Findeisen [29]). Some researchers, familiar with the hierarchical control literature, introduced these ideas in order to describe certain aspects of policy coordination within the European Union (see Ito and de Zeeuw [48]). In Ito et al. [49] a hierarchical model was designed and subsequently estimated. In this model the European Commission is modelled as a coordinator which adjust the actions of underlying countries in order to achieve an overall performance, whereas the countries may aim at their own objectives. A strong point of the model was the presence of the tension of national governments who's decision making is influenced by their own objectives and the Community objectives. With the model various experiments were undertaken which are reported in van der Wal et al. [79] and Weeren et al. [82]. However some main drawbacks of the model were also discovered. Besides some unconvenient model properties, the role of the European Commission in the model was rather extreme. Firstly, their available policy instruments were not in agreement with reality and secondly the role of the European Commission was rather strong in the sense that countries had no possibility to deviate from policy implications imposed by the Commission. In order to cope with these drawbacks the structure of the model needed to be changed. This, however, appeared to be an almost impossible task and therefore the hierarchical control research was abandoned in favour of the research presented in this thesis. In this thesis we still consider the tension between Community objectives and national objectives but the role of the EC will be modelled more flexible in the sense that in our concept it is possible to vary the impact of EC on the national economies. By choosing this framework we return to the more traditional views of policy coordination as described in the previous subsection. A recent study on modelling coordination issues in hierarchical control is Weeren [81].

1.4 Contents of the thesis

In this thesis the impact of convergence on coordination aspects are studied for Stage two of the EMU process. The restriction to consider only Stage two had to be made since we wanted to obtain empirical results with the help of an estimated multi-country model. It is clear that with such a model the welfare gains and losses if countries are already assumed to be in Stage three of EMU are very hard to grasp. Not only is Stage three relatively far away in time but also will Stage three be characterised by huge structural changes in the Community which are very difficult to predict. On the other hand, economic policy-making of the EU-economies in Stage two depends very much on the expected welfare gains in Stage three. If a EU-country expects to gain a lot from Stage three of EMU then that country is willing to accept more costs, in terms of welfare loss, in Stage two. Political reality suggests the same, however, sometimes arguments which put forward this point are negatively stated in the sense that they state that it will be very costly for those countries which will not enter Stage three of EMU. For instance, in the case of the United Kingdom, Ashdown [1] argues that staying out of the monetary union would have the main disadvantage that one isolates from the union and thus looses important economic and political influence. Such an isolation is likely to diminish trade, attract less foreign direct investment, increase exchange rate uncertainty and diminish influence in global politics. These arguments suggest also that the more countries in Europe participate at the European Union the harder it will be for those countries who stay outside. However, in the theoretical and empirical analysis in this thesis we will not consider the possible gains of Stage three, or differently stated the gains (or losses) for each country in Stage three is assumed to be zero. We will only briefly indicate how a part of the theoretical analysis will change if countries expect positive welfare gains from Stage three.

During Stage two of the EMU process it is assumed that the EC has some global objectives but has no direct control over policy instruments to reach them. This assumption fits reality probably quite closely since, as mentioned before, the only way for the EC to influence the Member States is to make (public) recommendations (see, e.g., the broad economic guidelines as published by the European Commission [16]). Although most countries positively react on these guidelines set out by the EC, the EC institutions themselves can only hope that each Member State will really act according these guidelines.

Now the main research of this thesis results from the question: How can we model the impact of decisions made by the European Commission on the underlying EU-economies? We consider the EC not only as a coordinating institution in the sense that it coordinates countries in order to establish Pareto efficient outcomes but there is more at hand. Firstly, it is, a priori, unclear whether the recommendations and guidelines set out by the Commission really coincide with the policy rules obtained as obtained in Pareto efficient outcomes. Secondly, one can even claim that certain rules set out by the EC may hamper welfare in Stage two, not only for some countries but also for the total union. For instance if we look more specificly at the four convergence conditions set out by the EC then each particular EU-country should raise itself the following question (see Crockett [17]): 'are the convergence rules consistent with a satisfactory performance in other domains of policy?'. For example, is low inflation not reached at the expense of low growth and high unemployment and is there sufficient structural flexibility in each economy to maintain a fixed exchange rate regime? It is clear that the answers on these question are different for each EU-economy. Some countries will find it harder to follow certain rules during Stage two than others. To study these aspects we model the EC-objectives as a seperate function. Constructing this function we use the convergence conditions as set out at the Maastricht Treaty 1991. It is clear that other (real) objectives, such as GDP per head or unemployment in the total union are respectable alternatives as well, but the convergence rules are more convenient for practical research since they are clearly specified.

The theoretical framework considered in this thesis is, therefore, as follows. Recall the situation just before the Maastricht Treaty 1991. Assume that each Member State enters Maastricht with an own objective (welfare) function in which they have specified their desired policy objectives and priorities for the coming years. We follow the traditional policy coordination theory, in which there are more than two players involved in the game and compare a situation where the EU-countries agree on a cooperative outcome, e.g. represented by a Pareto efficient outcome, versus a situation of a non-cooperative agreement, e.g., a Nash equilibrium.

Remark. In the case of more than two players this gives already some additional problems, such as coalition forming. In this thesis we refrain from coalitions and just consider two extreme regime types, a fully cooperative regime versus a fully non-cooperative regime, where the last regime implies that if one player decides to play non-cooperatively, all the players will do so. This assumption is made for convenience sake since the amount of coalitions, e.g., in the case of more than five countries is already very large.

The new contribution to the policy coordination literature is that besides these traditional game experiments we study the impact of conditions imposed on this game by a supranational institution. In theory we can distinguish two types of conditions (see Siebrand [71]):
(1) central conditions:
These are conditions which are imposed on the total economy such as in our example the EU, where the total economy is described by aggregate variables such as GDP per head in the EU, total employment in the EU, average price deflators etc. The central conditions we will use in this thesis are the two convergence conditions as specified in the Maastricht Treaty: convergence in nominal long term interest rates and convergence in consumer price deflators.
(2) decentral conditions:
Decentral conditions are conditions which are imposed by the supranational institution on

the national economies, where the national economy is described by national variables such as GDP per head or total employment in that particular economy. Examples in the Maastricht Treaty of decentral conditions are the two requirements for the national budgets. Remark that the main difference between these two types of conditions is that, in principle, the second type of conditions can be fulfilled by one particular economy through individual policymaking whereas the first type of conditions have to be fulfilled through collective policymaking. In this thesis we formalise these conditions imposed by the supranational institution by an EC-objective function. The introduction of this EC-objective function is interpreted as a restriction on the negotiation game of the EU-countries. Theoretically we can now distinguish two different views of policymaking.
Firstly, a cooperative point of view:
If we assume that countries play cooperatively then any restriction imposed on the game will decrease total welfare in comparison to the case where there is no restriction imposed on the game. Thus in this framework, if countries decide to put some weight on the EC-objectives they will envisage less room for policy-making concerning their own objectives. A priori, it is however not clear how total welfare losses are distributed among countries. For instance for some countries the Community objectives may coincide more with their own objectives than for others. Such outcomes depend for each individual country very much on its own structure of the economy and more in particular on the signs and sizes of the various international interdependencies of the economies.
Secondly, a non-cooperative point of view:
In a non-cooperative world the outcome of imposing restrictions on individual policymakers is unclear, it may be benificial but it may also be malicious for total welfare but also for each individual country. In comparison with a non-cooperative framework without EC-objectives this depends again on the structures of the underlying economies. It may be benificial if the EC-objectives are specified such that the size of the negative spillovers decreases and/or such that the size of positive spillovers increases. On the other hand it may be malicious if the size of positive spillovers decreases and/or the size of negative spillovers increases.

Thus from a non-cooperative point of view the outcome of the empirical experiment hinges decisively on the international interdependencies in the economies and the restrictions imposed on them, whereas in the cooperative framework the impact of restrictions is always malicious. Therefore, to study these aspects in the EU we have to get an idea of the (inter)dependencies of the various EU-economies (see for related subjects Hughes Hallett [35, 37]).

Remark. Weakening the assumption of zero profits or zero losses at Stage three could influence the game in a cooperative framework at least in two important ways. Firstly, if countries expect positive gains during Stage three then Pareto optimal policies during Stage two, which do not give a guarantee for a transition to Stage three of EMU, are not acceptable. In that case countries will search for policies which yield welfare gains which give a guarantee to enter Stage three of EMU and are not too far away from Pareto efficient solutions. This last argument is used in chapter two and explains why we do not consider in our analyses only policies on the Pareto frontier but also policies which yield a lot of convergence. Secondly, assume that the non-cooperative game outcome, where each country tries to minimise its own welfare without restrictions, is the threatpoint of the game. If now each country expects a positive amount of welfare gain in Stage three then one could argue that during Stage two a country would be willing to consider even (restricted) cooperative policies which yield less welfare gain than the welfare gain obtained at the threatpoint if this policy is a guarantee for entering Stage three.

This thesis can be divided in two parts, a theoretical one and an empirical one. The theoretical exposition is given in chapters two and three. In these chapters we study properties of the (restricted) cooperative game and related aspects from a cooperative point of view. The empirical part of this thesis is presented in chapters four and five. In chapter four we design a dynamic macro-economic model of eight EU Member States, the USA and Japan and in chapter five we compute (restricted) cooperative as well as (restricted) non-cooperative game outcomes for this model. The thesis is organised in such a way that each chapter is more or less self contained. Chapters two till four are preliminaries for chapter five. In that last chapter we report the empirical results obtained with the theory of chapter two and three and the estimated dynamic model of chapter four. In the following subsections we present a brief overview of the contents of each chapter separately.

1.4.1 Chapter two

This chapter is largely based on Douven and Engwerda [19] and presents a theoretical framework in which the functioning of the European Commission is studied from a cooperative point of view. In our dynamic game analysis the impact of the European Commission is modelled by formulating a convergence function. This aspect of convergence is modelled as a dynamic constraint on the individual loss functions. We show that if each individual country is not willing to accept a lower welfare than the welfare obtained in the threatpoint we have that the maximum degree of convergence is completely determined by this threatpoint. To illustrate the theory we present a (theoretical) two player example. The example shows that the costs involved to obtain convergence can differ substantially between countries and that, ultimately, these high costs can result in non-cooperative behaviour of both countries. Furthermore, it is shown that a small deviation from a Pareto optimal solution can increase convergence considerably. An algorithm is devised how to obtain solutions of the game which are politically more feasible than Pareto optimal solutions and improve on the non-cooperative solution.

1.4.2 Chapter three

This chapter is based on Douven and Engwerda [20]. We concentrate on Pareto optimal policies in the unrestricted cooperative game. We look at properties of N-person axiomatic bargaining solutions under the technical assumptions that the Pareto frontier is strictly concave and twice differentiable[1]. It contributes to the policy coordination literature in several ways. Firstly, a fast algorithm is given for computing the Nash bargaining solution, which is a well known solution in axiomatic bargaining theory. Secondly, we give empirical and theoretical arguments for the fact that the Nash bargaining solution and an other well known axiomatic bargaining solution, the Kalai-Smorodinsky solution, are often very closely situated on the Pareto frontier in policy coordination games. Thirdly, we consider effects of certain changes in the threatpoint, say $d = (d_1, ..., d_N)$, for the Nash bargaining as well as for the Kalai-Smorodinsky solution if the Pareto frontier remains fixed. If d_i increases, while for each $j \neq i$, d_j remains constant, then the corresponding Kalai-Smorodinsky solution has the property that player i is the only one who gains. This property is however not generally met for the Nash bargaining solution.

1.4.3 Chapter four

In order to compute empirically the theoretical implications of chapters two and three we need a macroeconomic multi-country model which contains various international linkages. In this chapter, which is a slightly extended version of Douven and Plasmans [22], we build a small linear interdependent model (SLIM) of eight EU-Member States (Belgium, Denmark, France, Germany, Ireland, Italy, the Netherlands and the United Kingdom), the USA and Japan. Since the Mundell-Fleming model became, and still remains, the starting point of open economy macroeconomics, we introduce in this chapter a modified version of this theoretical two-country framework. The Mundell-Fleming model is extended in three ways. Firstly, it is extended to more than two countries using the principal trading pattern of each

[1]Since this chapter is more related to the game theory literature, where games are defined in terms of maximising payoffs, we consider concave Pareto frontiers whereas in chapter two and five, where difference games are defined in terms of minimising costs, we consider convex Pareto frontiers. It is clear that there exists a homomorphism between the two frameworks since one can replace the payoffs by costs and maximisation by minimisation.

individual country. Secondly, we extended the model by including country-specific labour market characteristics, wage-price spirals and long term interest rates. Thirdly, we included dynamic responses into the model which make it possible to distinguish between short- and long-run behaviour of the economy. In each country direct linkages are modelled through outputs, prices, exchange rates and interest rates. For estimation we use annual data for the sample period 1960-1991. This estimation process is based on error-correction arguments. Historical simulations and shock analyses are performed to show various properties of the model and the outcomes of the model are compared with those for existing models in literature.

1.4.4 Chapter five

In chapter five we apply the theory developed in chapters two and three on the SLIM-model of chapter four. Starting from the Maastricht Treaty 1991, we assume that Stage two of EMU takes place during the period 1992-1999. We construct for this planning period welfare functions which describe the EU-countries' objectives and the EC-objectives. Next we compute and compare four possible scenarios. Two scenarios, a cooperative and a noncooperative, where the EU-Member states neglect the restriction imposed by the EC and two scenarios, a cooperative and a noncooperative, where the EU-Member States give some weight to the EC-objective function. In these last two scenarios the EU-Member States pursue a restricted policy. This restriction is modelled through the EC-objective function and represents conditions elaborated at the Maastricht Treaty. The chapter is based on Douven and Plasmans [21] and contains some limited overlap with previous chapters.

Chapter 2

Cooperative convergence outcomes

2.1 Introduction

Due to the integration process within the EU countries there is an increasing demand for price stability. To that end the European Council decided, at the Maastricht meeting in 1991, to start, at least in 1999, with irreversibly fixed exchange rates and to establish a European Central Bank. This final step towards the realisation of the EMU sets out, however, that uneven developments in the process of integration are set aside. Therefore, greater convergence of economic performance is needed (see the report of Delors Committee [18]). Another aspect of the integration process is that as a consequence of the strengthened economic interdependence between member countries the room for independent policy manoeuvre is reduced considerably and that cross-border effects of developments originating in each member country become increasingly important. So, the stages towards an economic and monetary union involve on the one hand a process of closer convergence, and on the other hand coordination of the macroeconomic policies of the various countries. Important to note is that this of course does not imply that if there is convergence of economic policies and/or coordination of macroeconomic policies between countries, the integration process will be balanced and thus the establishment of a single market is justified. In other words convergence and coordination are prerequisites for obtaining a single market but don't guarantee a succesful establishment of it. Now, there is a general consensus amongst the participating countries that convergence and coordination of policies is needed for moving towards an economic and monetary union. There is, however, much less consensus how far and how fast this process should take place. This has, of course, everything to do with the internal forces working on the markets of each individual country. The possibly long run significant increases in economic welfare in the Community are much less tangible than the short term welfare losses incurred at various domestic markets. Therefore, a natural reaction one can expect from participating countries is that they do strive for convergence in economic variables, but that they are only willing to pay a price (in terms of welfare loss) for it if the additional increase in the degree of convergence will be significant. Studies with respect to macroeconomic policy coordination in a dynamic games context appear frequently in economic literature, see e.g. Brandsma [8], McKibbin and Sachs [55], Hughes Hallett [39]. However, the influence of the aspects of convergence, analysed in a dynamic games setting, on the effects of macroeconomic policy coordination are not studied before.

Starting from the point of view that each country has its own individual social welfare function it wants to minimise in cooperation with the other countries, we develop a theoretical framework to analyze the trade off between extra welfare loss and more convergence. The analysis will be done in a dynamic games framework. We assume that each policymaker has an individual objective function, he/she wants to minimise and that there is some common sense on a convergence function which they want to minimise simultaneously. In the case of the EMU this convergence function may e.g. represent the convergence conditions which are specified in the Maastricht treaty (1991). In particular the two conditions of convergence in consumer price inflation and convergence in long term interest rates that are necessary for admitting a country to the monetary union (see e.g. Bean [4]) can be incorporated in such a function. Under the assumption that all policymakers like to cooperate, we analyze the set of solutions which are obtained by the policymakers when they simultaneously minimise their welfare loss functions and convergence function. We assume that the degree of convergence, which is represented by the value of the convergence function, depends on the agreements of the outcome of a negotiating process between countries. In particular we will show that if reducing welfare loss is the primary interest of countries, the degree of convergence countries can obtain is limited. So, if countries strive for a degree of convergence which is set too ambitious, the result can be that (some) countries will show non-cooperative behaviour. In a theoretical example we illustrate two additional aspects the game may have.
(1) The price (in terms of individual welfare loss) that countries have to pay will for some countries be higher than for other countries.
(2) There are situations in which by a minor deviation from the Pareto solution, a large increase in convergence degree is possible. In other words, by paying a small price (in terms of individual welfare loss) high revenues (in terms of convergence) can be obtained.

The organisation of the chapter is as follows. In section 2.2 we will introduce the theoretical framework. We consider N countries which cooperatively agree on minimising a convergence function and, moreover, all have their own individual objective function they like to minimise. The aspect of convergence is modeled as a dynamic constraint on the joint social welfare function. Under the assumption that all of these functions are convex and (some mild regularity conditions) we show the above mentioned aspects. Furthermore we show that the cooperative outcome which yields the largest degree of convergence coincides with the Nash solution of the game. To help the reader to understand the basics of the presented theory we illustrate the approach in section 2.3 by means of a simple theoretical example. In section 2.4 we present the conclusions.

2.2 Incorporating convergence criteria: a theoretical framework

We consider an integrated economy of the European Community with N interdependent economies, where the policymakers in each country face a dynamic economic model which connects the endogenous variables (denoted by y), instrumental variables (denoted by u) and other noncontrollable variables. Each country has control over a set of instruments for economic policy, denoted by u_i. In stacked form $u' = (u_1', ..., u_N')$. We assume that each policymaker has a convex objective function, which we specify by J_i, which he/she wants to minimise. We denote the set of Pareto optimal solutions in the $J_1, ..., J_N$-plane by P. The point N_c corresponds to the non-cooperative (Nash) solution, which is used as a bargaining threat-point, denoted by $N^c := (J_1^N, ..., J_N^N)$. Furthermore we assume that the countries agree to strive for a certain amount of degree of convergence for some of their economic (endogenous and/or instrumental) variables. This agreement will be reflected in a convex convergence function, denoted by C, which is included in the optimisation process. It is important to stress that the convergence function differs from the countries objective functions in a way that the latter contains only variables which belong to its own country whereas the convergence function contains variables of all the countries. Thus minimising a costfunction is something that can, in principle, be done by a country alone whereas minimising the convergence function has to be done simultanously.

The decision-making process of the policymakers concerning what strategy to follow, will depend on the following set:

$$\{(J_1(u), ..., J_N(u), C(u)) \mid u \in U\}, \tag{2.1}$$

where we suppose that the strategy-space U is a convex set. The policymakers have to find a cooperative strategy which results in a point in (2.1) which is acceptable for them all. Now note that whenever two different strategies yield the same individual costs $J_i, \quad i = 1, ..., N$, but different values for the convergence function, only the strategy yielding the lowest value for the convergence function is of interest to all policymakers. So, the set of relevant control strategies consists of:

$$\bar{U} = \{u \in U \mid \forall \bar{u} \in U \ \ (J_1(u), ..., J_N(u)) = (J_1(\bar{u}), ..., J_N(\bar{u})) \Rightarrow C(u) \leq C(\bar{u})\}.$$

This observation makes it possible to consider the decision problem from the following point of view. By varying the strategies over the whole set $\bar{U}$, we obtain the set of all possible objective outcomes in the $J_1, .., J_N$-plane. To each point in this set is attached a unique value for the convergence function. The problem for the decision makers is now to select cooperatively a point within this set that is acceptable for everyone. Now, as mentioned in the introduction we will assume that minimising their own cost function is the primary interest of countries and that striving for convergence is of secundary interest. In that case the aspect of convergence acts as a dynamic constraint on joint social welfare. If we, furthermore, refrain from the possibility of side-payments and assume that the axiom of individual rationality holds (see e.g. Petit [66] [1]), then countries will cooperatively minimise the joint convergence function as long as their individual costs will be lower than their non-cooperative costs. So, the set of possible objective outcomes will then restricted on the one hand by the non-cooperative Nash threatpoint N^c, and on the other hand by the set of Pareto solutions. We will call this set in the sequel the negotiation area (see figure 2.1 for an illustration in a two player context).

Remark. To complicate matters not unnecessarily we, here, do not address the issue of information patterns and period of commitment (see Basar and Olsder [3]). For explaining our ideas it is sufficient (and most convenient) to use an open-loop information structure and binding commitments, which fixes the 'negotiation area' throughout the entire planning period. In the closed-loop case, we have to take account of multiple (Nash threatpoint) equilibria and if we also take account of the possibility of renegotiation our 'negotiation area' would vary over time. In the case of multiple equilibria, in literature various kind of arbitration schemes and algorithms have been proposed to discriminate between these equilibria. An overview of the literature can be found in de Zeeuw and Hughes Hallett [86, 36]. We use the convergence-criteria as an arbitration scheme to discriminate between the cooperative points. However, when introducing such an arbitration scheme points outside the negotiation set (i.e., the Pareto optimal solutions between A and B in figure 2.1) become interesting too. This is what we will investigate in the sequel.

The basic question is now of course, how we can determine this negotiation area and

[1]This axiom states that policymakers, if they behave rational, will never accept an outcome for their individual object function which is worse than the one a policymaker can obtain by acting independently (which is represented by the non-cooperative outcome N^c).

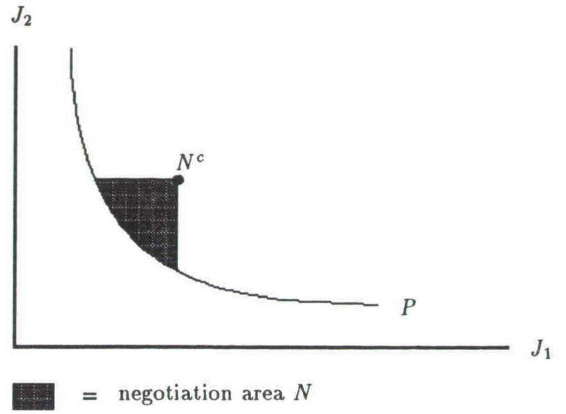


Figure 2.1: Representation of the negotiation area in a two player context.

its corresponding convergence function values in an efficient way. We will not give a complete answer to this question, but present a solution which we expect will work for the applications we are aiming at (i.e. situations in which the set of Pareto-solutions and the Nash-threatpoint are situated not too far from each other). The solution we will present has a number of nice properties. First of all it attaches to every point in the negotiation area a unique control strategy that can be obtained by minimising a strict convex combination of the individual object functions and the convergence function. Secondly, we will show that this control strategy is parametrised by N parameters and that this parametrisation is a continuous function of its parameters. By varying the parameters between 0 and 1, the whole negotiation area can then be covered (in general (see note above)).

The solution is motivated by our assumption that each policymaker is primarily interested in minimising his own objective function in a cooperative setting and that convergence plays a minor role. We model this aspect by rewriting the convex combination of individual cost and convergence cost in a special way. Consider

$$\bar{\alpha}_1 J_1 + ... + \bar{\alpha}_N J_N + \bar{\alpha}_{N+1} C, \text{ with} \sum_{i=1}^{N+1} \bar{\alpha}_i = 1.$$

This is equivalent with (in the non-trivial case $\bar{\alpha}_{N+1} \neq 1$):

$$(1-\lambda)(\alpha_1 J_1 + ... + \alpha_N J_N) + \lambda C, \text{ where } \lambda = \bar{\alpha}_{N+1}, \text{ and } \alpha_i = \bar{\alpha}_i/(1-\bar{\alpha}_{N+1}),$$

which has the nice property that $\sum_{i=1}^{N} \alpha_i = 1$. If we minimise this second convex combination of the individual object functions and the convergence function then we have the property that $\lambda = 0$ resembles the case that countries completely ignore the convergence goal (and because $\sum_{i=1}^{N} \alpha_i = 1$ we find the Pareto optimal solutions), and that $\lambda = 1$ corresponds with the case that countries only pay attention to their mutual convergence interests. We will show (under some smoothness conditions) that the set of cooperative optimal strategies corresponding with these adapted object functions for each of the N countries, can be parametrised by the $N-1$ parameters $\alpha_1, ..., \alpha_{N-1}$ and λ, and that this parametrisation is a continuous differentiable function of all these parameters. By varying these parameters, in particular λ, it is then possible to analyze the trade off between the costs individual countries have to pay and more convergence. First, we present a preliminary result. The next theorem shows that if one considers a certain convex combination of all object functionals J_i, $i = 1, .., N$ and C, the optimal strategy minimising this combination will be a continuous differentiable function of N out of $N+1$ parameters.

Theorem 2.1 *Suppose U is a convex set, $J_i(u), i = 1, ..., N$ and $C(u)$ are strictly convex functionals which are twice continuously differentiable in $u \in U$. Consider*

$$J(u, \alpha_1, ..., \alpha_N, \lambda) := (1-\lambda)\left(\sum_{i=1}^{N} \alpha_i J_i(u)\right) + \lambda C(u)$$

for $u \in U$, $\lambda \in [0,1]$ and $\alpha_i \in [0,1]$ for $i = 1, ..., N$, with $\sum_{i=1}^{N} \alpha_i = 1$. Let

$$u^* := \arg\min_u J(u, \alpha_1, ..., \alpha_N, \lambda).$$

Then, for every $\lambda \in [0,1]$ and $\alpha_i \in [0,1]$ for $i = 1, ..., N$, with $\sum_{i=1}^{N} \alpha_i = 1$, u^* is uniquely determined as a function of the parameters $\alpha_1, ..., \alpha_{N-1}, \lambda$, i.e. $u^* = u^*(\alpha_1, ..., \alpha_{N-1}, \lambda)$. Moreover, this function u^* is a continuously differentiable function in $(\alpha_1, ..., \alpha_{N-1}, \lambda) \in [0,1] \times ... \times [0,1]$, with $\sum_{i=1}^{N-1} \alpha_i \leq 1$.

Proof. Let $\bar{\alpha} := (\bar{\alpha}_1, ..., \bar{\alpha}_N, \bar{\lambda}) \in [0,1] \times ... \times [0,1]$ be fixed numbers, with $\sum_{i=1}^{N} \bar{\alpha}_i = 1$. The strictly convex properties of $J_1, ..., J_N, C$ imply that the function $J(u)$ is strictly convex in $u \in U$. So for every $\bar{\alpha}$, J has a unique global minimum on U. Denote this element in U, which depends on $\bar{\alpha}$, by $u_{\bar{\alpha}}$. Since J is differentiable we conclude that the derivative of J with respect to u, evaluated at the point $u_{\bar{\alpha}}$ is zero. So,

$$F(\bar{\alpha}_1, ..., \bar{\alpha}_N, \bar{\lambda}, u) := \frac{\partial J(u)}{\partial u} = (1-\bar{\lambda})\bar{\alpha}_1 \frac{\partial J_1(u)}{\partial u} + ... + (1-\bar{\lambda})\bar{\alpha}_N \frac{\partial J_N(u)}{\partial u} + \bar{\lambda}\frac{\partial C(u)}{\partial u} = 0$$

evaluated at the point $u = u_{\bar{\alpha}}$. Note that, since J is by assumption twice continuous differentiable, the functional F is continuous differentiable in $(\alpha_1, ..., \alpha_{N-1}, \lambda, u) \in [0,1] \times ... \times [0,1] \times U$. Furthermore, since $J_1, ..., J_N, C$ are strictly convex functionals in u, we have that

$$\forall \bar{\alpha} \in [0,1] \times ... \times [0,1] \quad det \frac{\partial F(\bar{\alpha}, u_{\bar{\alpha}})}{\partial u} \neq 0.$$

Applying the implicit function theorem yields then that there is an unique continuous differentiable function, say f, such that for all $\alpha := (\alpha_1, ..., \alpha_N, \lambda) \in [0,1] \times ... \times [0,1]$, $F(\alpha, f(\alpha)) = 0$, with $f(\alpha) = u_\alpha$. So, $u^* := u^*(\alpha_1, ..., \alpha_N, \lambda) := f(\alpha_1, ..., \alpha_N, \lambda)$ is a continuous differentiable function in $\alpha \in [0,1] \times ... \times [0,1]$. Using the fact that $\sum_{i=1}^N \alpha_i = 1$ gives $u^* = u^*(\alpha_1, ..., \alpha_{N-1}, \lambda)$. □

Remark. In the sequel we will use the notation $(\alpha_1, ..., \alpha_{N-1}) \in [0,1] \times ... \times [0,1]$, but, by doing so, we implicitly assume that the $\alpha_i, i = 1, ..., N-1$ satisfy the constraint $\sum_{i=1}^{N-1} \alpha_i \leq 1$.

Using the previous result we show now that the set of control strategies defined in theorem 2.1, parametrised by

$$\bar{\bar{U}} := \{u^*(\alpha_1, ..., \alpha_{N-1}, \lambda), \mid (\alpha_1, ..., \alpha_{N-1}, \lambda) \in [0,1] \times ... \times [0,1]\}$$

has the advertised properties. Formally the result reads as follows:

Theorem 2.2 *There exists a bijective mapping between the set of unique points*

$$\{u^*(\alpha_1, ..., \alpha_{N-1}, \lambda) \mid (\alpha_1, ..., \alpha_{N-1}, \lambda) \in [0,1] \times ... \times [0,1]\}$$

and the set

$$\{(J_1(u^*), ..., J_N(u^*), C(u^*)) \mid (\alpha_1, ..., \alpha_{N-1}, \lambda) \in [0,1] \times ... \times [0,1]\}.$$

Furthermore $J_1(u^*), ..., J_N(u^*), C(u^*)$ are continuous functions in $(\alpha_1, ..., \alpha_{N-1}, \lambda) \in [0,1] \times ... \times [0,1]$.

Proof. Because $(1-\lambda)(\sum_{i=1}^N \alpha_i) + \lambda = 1$, with $\lambda \in [0,1]$ and $\alpha_i \in [0,1]$,for $i = 1, ..., N$, the unique solution u^* of $J(u)$ is a Pareto solution for the objective function $J(u)$ which represents a game with $N+1$ players, where each player minimises the objective function represented by J_i for player i, $(i = 1, ..., N)$ and C for player $N+1$. According to ,e.g., de Zeeuw [85, lemma 3.4.2 and 3.4.3] there is a bijective mapping between the Pareto solutions for $J_1, ..., J_N, C$ and the optimal solution for J. The set of Pareto solutions can be found by varying the parameters $(\alpha_1, ..., \alpha_N, \lambda)$ between $[0,1] \times ... \times [0,1]$ with $\sum_{i=1}^N \alpha_i = 1$. Because u^* is a continuous function in $(\alpha_1, ..., \alpha_{N-1}, \lambda) \in [0,1] \times ... \times [0,1]$ it is straightforward that $J_1(u^*(\alpha_1, ..., \alpha_{N-1}, \lambda)), ..., J_N(u^*(\alpha_1, ..., \alpha_{N-1}, \lambda)), C(u^*(\alpha_1, ..., \alpha_{N-1}, \lambda))$ are continuous functions in $(\alpha_1, ..., \alpha_{N-1}, \lambda) \in [0,1] \times ... \times [0,1]$. □

Using the theorem, the set of control strategies $\bar{\bar{U}}$ gives us the following subset of (2.1):

$$\{(J_1(u^*), ..., J_N(u^*), C(u^*)) \mid u^* \in \bar{\bar{U}}\} \tag{2.2}$$

To see that this reduction of the set in (2.1) still contains all the interesting points, we analyze the set in (2.2) in combination with J more specifically. We have that:
(i) the set in (2.2) contains the whole set of points $(J_1, ..., J_N)$ which belong to the Pareto optimal solutions. To find these solutions we substitute $\lambda = 0$ in $\bar{\bar{U}}$ and fill in the resulting control strategies in (2.2).
(ii) the set in (2.2) contains the points where C is minimal. To find these points we substitute $\lambda = 1$ in $\bar{\bar{U}}$ and fill in the resulting strategies in (2.2).
Furthermore, from theorem 2.2, we have that the set of points in (2.2) form a continuous surface in the $J_1, ..., J_N, C$-plane, which indicates that we have parametrised all the interesting points between (i) and (ii) as well. These points can be found by varying λ between 0 and 1.

From now on we will skip the u^* in the notation and describe the set in (2.2) as:

$$\{(J_1, ..., J_N, C) \mid (\alpha_1, ..., \alpha_{N-1}, \lambda) \in [0,1] \times ... \times [0,1]\}. \tag{2.3}$$

We will now define some sets of interesting points. A projection of the set in (2.3), on the $J_1, ..., J_N$-plane is:

$$S := \{(J_1, ..., J_N) \mid (\alpha_1, ..., \alpha_{N-1}, \lambda) \in [0,1] \times ... \times [0,1]\}$$

The subset of S:

$$P := \{(J_1, ..., J_N) \mid (\alpha_1, ..., \alpha_{N-1}, 0) \in [0,1] \times ... \times [0,1]\}$$

represents the set of Pareto solutions. Iso-convergence lines, i.e. lines with the same degree of convergence, are defined as follows:

$$I_\gamma := \{(J_1, ..., J_N) \mid C(\alpha_1, ..., \alpha_{N-1}, \lambda) = \gamma, \quad (J_1, ..., J_N) \in S, \quad \gamma \in \mathbb{R}^+\}$$

Note that a small value of γ corresponds with much convergence (and vice versa). The negotiation area is defined by:

$$\mathcal{N} := \{(J_1, ..., J_N) \mid J_1 \le J_1^N, ..., J_N \le J_N^N, \quad (J_1, ..., J_N) \in S\}$$

Using the axiom of individual rationality it is clear that policymakers will not agree to a certain degree of convergence, denoted by γ, if $I_\gamma \cap \mathcal{N} = \emptyset$. Moreover, the largest degree of convergence policymakers are willing to accept is given by:

$$\gamma^* := \min_\gamma \{\gamma \mid I_\gamma \cap \mathcal{N}\} \neq \emptyset.$$

So, in general policymakers should set their degree of convergence with care because if this degree is set too ambitious policymakers are not willing to cooperate anymore. In the next theorem we will prove fact that the point in the negotiation area which yields the largest degree of convergence is the non-cooperative threat point N^c. This (unique) point in (2.3) will in the sequel be denoted by C^{max}.

Theorem 2.3 *If $\mathcal{N} \subset S$ then the point in the negotiation area $\mathcal{N}$, represented by a $x \in \bar{\bar{U}}$, for which $C(x) = \gamma^*$ equals N^c.*

Proof. According to Theorem 2.2., there is a bijective relationship between $\bar{\bar{U}}$ and the set of Pareto solutions which correspond to a game of $N+1$ players, where player i, $(i = 1, ..., N)$, minimises an objective function represented by J_i, and player $N+1$ minimises C. Suppose that $u \in \bar{\bar{U}}$ yields a point in S which lies in the negotiation area $\mathcal{N}$ which differs from N^c but for which convergence is maximal. Since u yields a point $(J_1(u), ..., J_N(u))$ in the negotiation area it satisfies the property that $J_i(u) \leq J_i^N$. Because the strategy u corresponds with a point in S that differs from N^c, there is an $i \in 1, ..., N$ for which $J_i(u) < J_i^N$. Making use of the angular shape of $\mathcal{N}$ and the assumption $\mathcal{N} \subset S$, it is now always possible to find a strategy $v \in \bar{\bar{U}}$ which corresponds with a point in S which lies on the boundary of $\mathcal{N}$ and for which $J_1(v) = J_1(u),...,J_i(v) = J_i^N,...,J_N(v) = J_N(u)$. Comparing these two points in the negotiation area, we have that of all the J_j-values $(j = 1, ..., N)$, only the J_i-value of the two points differ. Due to the fact that u and v are both Pareto optimal solutions it follows from the definition of Pareto optimality that the convergence value in both points differs as well. This observation implies that $C(v) < C(u)$. The fact that strategy v corresponds with a lower convergence value than u, violates the assumption that u corresponds with a point in $\mathcal{N}$, for which convergence is maximal. □

It is important to note at this point that the non-cooperative strategy which results in the point $N^c \in S$ in general differs from the cooperative strategy which results in the point C^{max}. In general, the convergence outcome of the non-cooperative strategy and the cooperative strategy will differ in the sense that the convergence value for the cooperative strategy will be lower than the convergence value for the non-cooperative strategy. So, the gains in convergence policymakers will receive by playing cooperatively will be at most $\gamma^* - C(u_{N^c})$, where u_{N^c} represents the non-cooperative strategy which yields N^c. Thus, in general, Pareto solutions will not yield maximal convergence. Therefore, if policymakers want a certain degree of convergence, it will usually not be possible to keep up with the Pareto optimal solutions. Usefull Pareto solutions will only coincide with solutions with a certain degree of convergence, say γ, if $I_\gamma \cap \mathcal{N} \cap P \neq \emptyset$. Note, furthermore, that the price to be payed for reaching convergence of a certain degree will not be the same for every country. We will illustrate this in an example in the next section. If $\mathcal{N} \not\subset S$, the threatpoint is not guaranteed to fall within S. In that case our approach will not work, because we can not calculate all the points within the negotiation area. However, it is our experience that $\mathcal{N} \subset S$ will apply in most applications [2].

[2]Counter examples can be constructed by introducing erratic convergence functions or specifying dynamics for which the Pareto solutions and the Nash threatpoint are situated very far from each other.

2.3 An illustrative example

We consider a theoretic example in a (discrete time) deterministic linear quadratic difference game framework with two players (countries). The dynamic behaviours of player 1 and player 2 are described by:

$$\begin{aligned} y_1(t) &= y_1(t-1) + u_1(t) + 0.3y_2(t-1), \qquad & y_1(0) = 1, \\ y_2(t) &= y_2(t-1) + u_2(t) + 0.6y_1(t-1), \qquad & y_2(0) = 0, \end{aligned}$$

where, for $i = 1, 2,\ \ y_i(t) \in \mathbb{R}$ is the target variable and $u_i(t) \in \mathbb{R}$ is the instrumental variable. From the interaction terms ($0.3y_2(t-1)$ for player 1 and $0.6y_1(t-1)$ for player 2) follows that each player faces a different dynamical structure. Player 2 is more influenced by player 1 than vice versa. Each player makes his plans for the future. We assume that each player has a planning period of 2 periods and chooses his desired paths for the future, as follows:

$$\text{desired paths} \begin{cases} \text{player 1:} & y_1^*(1) = 2, \quad y_1^*(2) = 3 \\ \text{player 2:} & y_2^*(1) = 1.5, \quad y_2^*(2) = 3. \end{cases}$$

These desired paths reflect the policymakers own wishes of the future and are obtained independently from each other. In this example the players have different preferences but, as can be seen from the ideal paths, both players are striving for convergence of their target variables in period 2. It is of course not necessary to choose desired paths wich converge but by doing so we will be able to demonstrate the fact that Pareto optimal solutions do not coincide with convergence solutions, even if policymakers strive for convergence in their desired values. We represent the cost functions J_1, J_2 for every individual player by:

$$\begin{aligned} J_1 &= 0.5((y_1(1) - 2)^2 + (y_1(2) - 3)^2 + u_1(1)^2 + u_1(2)^2), \\ J_2 &= 0.5((y_2(1) - 1.5)^2 + (y_2(2) - 3)^2 + u_2(1)^2 + u_2(2)^2). \end{aligned}$$

Each player wants to play a strategy, during his planning period, which minimises his costs. So the control problem for every individual player ($i = 1, 2$) is:

$$\min_{u_i(1), u_i(2)} J_i.$$

Because the target variable (and indirectly the instrumental variable) of each player is directly related to those of the other player, the control problem of each player depends on the actions undertaken by the other player. This gives rise to various solution concepts. From the non-cooperative solutions we will just consider the open loop Nash solution, which we denote by N^c. The cooperative solutions are represented by the set of Pareto solutions which can be found by solving:

$$\min_{u} \alpha J_1 + (1 - \alpha) J_2.$$

for $\alpha \in [0,1]$, where $u := (u_1(1), u_1(2), u_2(1), u_2(2))$.

However, before playing the game both players want to be sure that there will be some degree of convergence of their target variables. In this example we assume that both players want to converge to the average of their target variables. We take as a measure for the degree of convergence the following convergence function:

$$C = \sum_{i=1}^{2}(y_i(1) - \overline{y}(1))^2 + 4(y_i(2) - \overline{y}(2))^2,$$

where $\overline{y}(t) := 0.5(y_1(t) + y_2(t))$ for $t = 1, 2$. So, both players agree that they want to minimise the variance of their target variables in each period. Moreover, minimising the variance in period 2 is given more weight than minimising the variance in period 1, which is represented by the weights of 1 in period 1 and 4 in period 2. These weights indicate that both players find it more important that there is convergence at the end of the planning period than during the planning period[3].

Now, together, the players have to take a decision about the strategy they are going to follow. In order to choose a strategy they have to weigh out all possible strategies. So, ultimately they have to find a strategy which is 'optimal' in some sense. In the next subsection we demonstrate the solution concepts developed in section 2.2 and analyze the space of interesting outcomes. After that we give one possible interpretation of 'optimal' and give a proposal to determine a feasible degree of convergence, γ, for both players.

2.3.1 Analysis of the possible outcomes

As stressed in section 2.2, the decision about what strategy to follow, will depend upon the following set:

$$\{(J_1(u), J_2(u), C(u)) \mid u \in \mathbb{R}^4\} \tag{2.4}$$

Because J_1, J_2, C are strictly convex functions which are twice differentiable in u, the set $\bar{\bar{U}}$ can be found by solving the following problem:
Let $\alpha, \lambda \in [0,1]$, and

$$J(u) := (1-\lambda)(\alpha J_1 + (1-\alpha)J_2) + \lambda C$$

[3]The above mentioned convergence criterium is in our two player case the same as minimising the quadratic sum of the differences between the target values: $C = 0.5(y_1(1) - y_2(1))^2 + 2(y_1(2) - y_2(2))^2$. Furthermore, note, that for convenience sake we not included control strategies in the convergence function which would make the problem indefinit. If one wants to use our approach for practical purposes, one should realise that scaling of the object functions and convergence function might be necessary.

Find now for every $\alpha, \lambda \in [0,1]$:

$$u^* := \arg\min_u J(u)$$

From section 2.2, the set of control strategies $\bar{\bar{U}}$ is given by:

$$\{u^*(\alpha,\lambda) \mid (\alpha,\lambda) \in [0,1] \times [0,1]\}$$

Substituting these control strategies in (2.4) gives the following set (compare with (2.3)):

$$\{(J_1(\alpha,\lambda), J_2(\alpha,\lambda), C(\alpha,\lambda)) \mid (\alpha,\lambda) \in [0,1] \times [0,1]\}. \tag{2.5}$$

In the sequel we will analyse this set of points for the given example.

Remark. Computing the outcomes for $\lambda = 1$, $\alpha = 0$, $\alpha = 1$ gives some difficulties because in that case we have a singular system of equations. However, we are not particularly interested in those situations so we used in our calculations values which are close to these points.

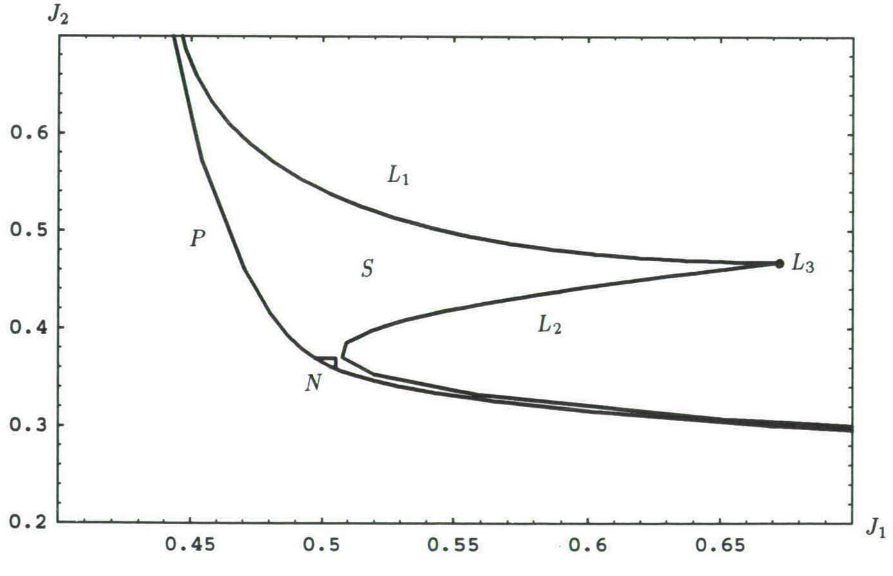


Figure 2.2: The parametrised area S, the most left curve represents the Pareto solutions P, the small triangle on this curve represents the negotiation area $\mathcal{N}$.

A projection of the surface in (2.5), on the J_1, J_2-plane is drawn in figure 2.2. This set of points is denoted by S, like in section 2.2.

$$S = \{(J_1(\alpha, \lambda), J_2(\alpha, \lambda)) \mid (\alpha, \lambda) \in [0,1] \times [0,1]\}$$

The black lines in figure 2.2 represent the edges of S. One of these edges is the set of Pareto solutions, which is given by the left black line. It is obtained by computing for various α:

$$P = \{(J_1(\alpha, 0), J_2(\alpha, 0)) \mid \alpha \in [0,1]\}$$

Points on the upper part of the Pareto line correspond with a high value of α and points on the lower part to a low value of α. The edge L_1 in figure 2.2 is obtained by computing for various $\lambda \in [0,1]$: $(J_1(1,\lambda), J_2(1,\lambda))$ and the edge L_2 by computing for various $\lambda \in [0,1]$: $(J_1(0,\lambda), J_2(0,\lambda))$. The edge in the figure which corresponds to $(J_1(\alpha,1), J_2(\alpha,1))$ for $\alpha \in [0,1]$ is reduced to one point in the figure. We denoted this point by L_3. The small triangle on the Pareto line denotes the negotiation area $\mathcal{N}$ as defined in section 2.2. Note that the negotiation area $\mathcal{N}$ is completely covered by S.

To get an idea of the degree of convergence in every point of the set in (2.3) we plotted figure 2.3. This figure shows a three dimensional plot of the following surface:

$$\{C(\alpha, \lambda) \mid (\alpha, \lambda) \in [0,1] \times [0,1]\}$$

Note that the points where $\lambda = 0$ give the degree of convergence for the Pareto solutions. As can be seen in the figure, points on the Pareto line where α almost equals 1 or 0, give a very high C value, which indicates that in those points the degree of convergence is rather small. Moreover we marked in this figure the point with the largest degree of convergence on the Pareto line. It is denoted by D and it corresponds with $\alpha = 0.836$. In table 2.1 the corresponding (J_1, J_2, C) of point D is given. Moving away from the Pareto

	J_1	J_2	C	α	λ
Cooperation					
C^{max}	0.505	0.370	0.1059	0.080	0.270
A	0.497	0.370	0.1258	0.625	0
B	0.505	0.359	0.1483	0.523	0
D	0.472	0.451	0.0896	0.836	0
Non-Cooperation					
N^c	0.505	0.370	0.1365	-	-

Table 2.1: Characteristics of some interesting points.

line, by increasing λ just a little bit, we see that the degree of convergence increases also.

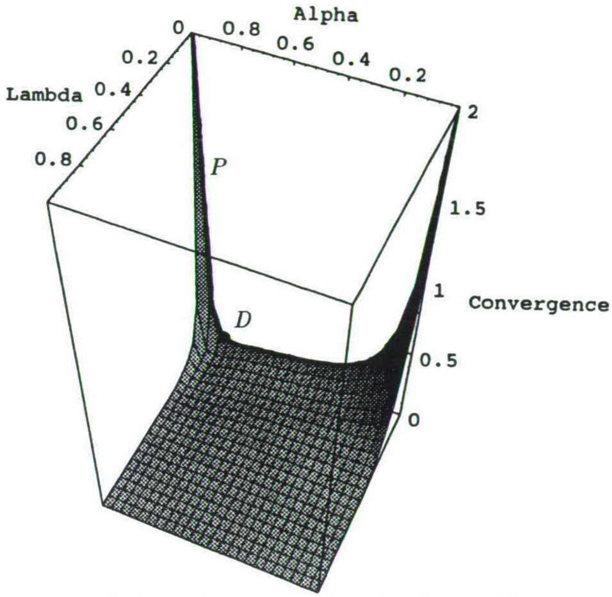


Figure 2.3: A three dimensional plot, where for each $\alpha, \lambda \in [0,1] \times [0,1]$ the corresponding convergence outcome is plotted. The curve on the back, where $\lambda = 0$, represents the Pareto solutions P.

This means that if the players choose an outcome outside the Pareto line P their costs will increase but in return for these increasing costs the value of the convergence function will decrease, which means more convergence. In this example, if λ increases to 1, the convergence function will go to 0, which means complete convergence (in period 1 and period 2). Zooming in on figure 2.2, around the negotiation area $\mathcal{N}$, gives us figure 2.4. Specific information about the points A, B, C^{max} and N^c can be found in table 2.1. In table 2.1 the cooperative outcome C^{max} corresponds with a strategy which yields a lower convergence value than the non-cooperative outcome N^c. The reason for this is that the non-cooperative strategy differs from the cooperative strategy. Playing the cooperative strategy gives, with the same individual costs, an increase in convergence of 0.306! In figure 2.4 we draw some iso-convergence lines, as defined in section 2.2. In the figure for each iso-convergence line the corresponding convergence value is given. The degree of convergence on the Pareto line increases from B to A. As proven in section 2.2 and visible in the figure, the point with the largest degree of convergence C^{max} lies on the edge of the negotiation area and coincides exactly with the N^c point which belongs to the iso-convergence line $I_{0.1059}$. So, the γ^*, as defined in section 2.2, equals 0.1059.

Zooming in on figure 2.3, from a different viewpoint, we get an indication of which values

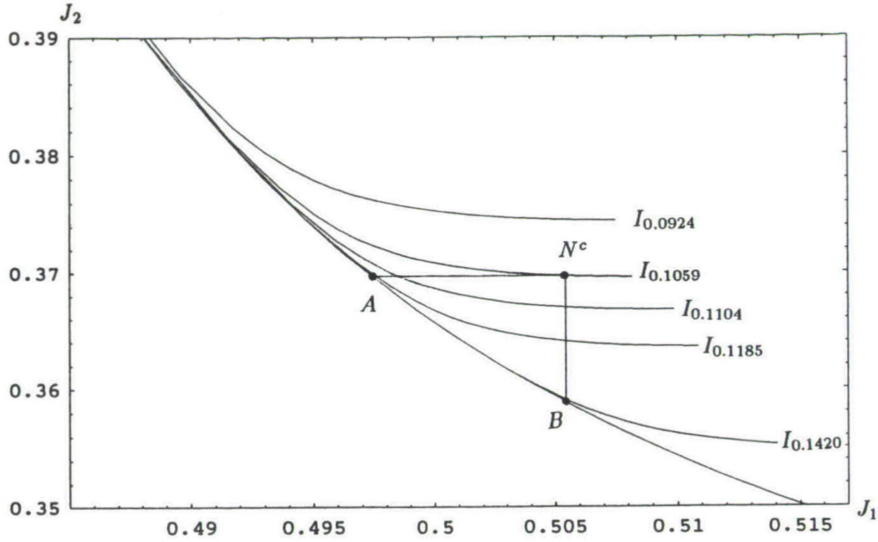


Figure 2.4: Zooming in around the negotiation area. Iso-convergence lines are drawn.

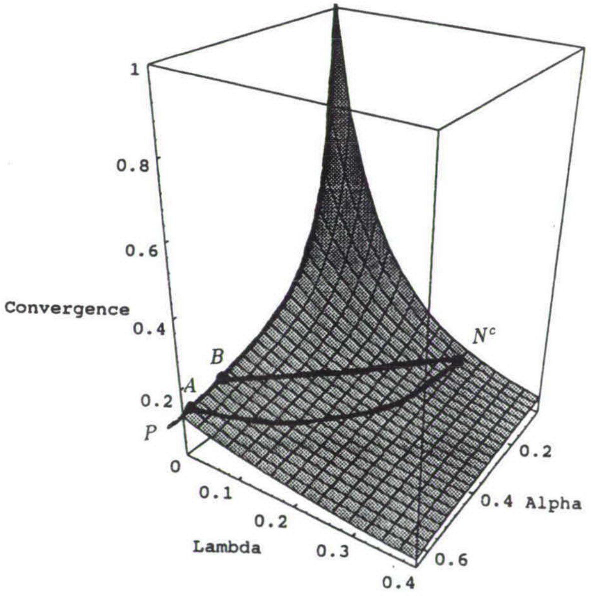


Figure 2.5: Zooming in around the negotiation area $(0.05 \leq \alpha \leq 0.65)$ and $(0 \leq \lambda \leq 0.4)$. The curve on the back (λ=0) represents a subset of the Pareto solutions P. The interior of the curve drawn on the surface represents the negotiation area $\mathcal{N}$.

of α, λ belong to the negotiation area $\mathcal{N}$, which is drawn on the surface in figure 2.5. The corresponding α, λ-values for A, B, and N^c are given in table 2.1. As one notes, the convergence value declines (so convergence increases) as λ increases.

2.3.2 Fixing the degree of convergence

In this subsection we assume that both players agree they want a degree of convergence of at least γ. So, the players will play a strategy which results in a point in (2.4) which belongs to the iso-convergence line I_γ. In figure 2.6 we have drawn for three different values of λ the convergence values for all $\alpha \in [0, 1]$. On the one hand it gives an indication of how quickly convergence declines when increasing λ, and on the other hand it illustrates how the convergence depends on α for constant λ. Again, $\lambda = 0$ corresponds with the Pareto optimal strategies. Moreover, in figure 2.6 four different levels for γ are drawn. Each level distinguishes a group of solutions with different properties which we will analyze below.

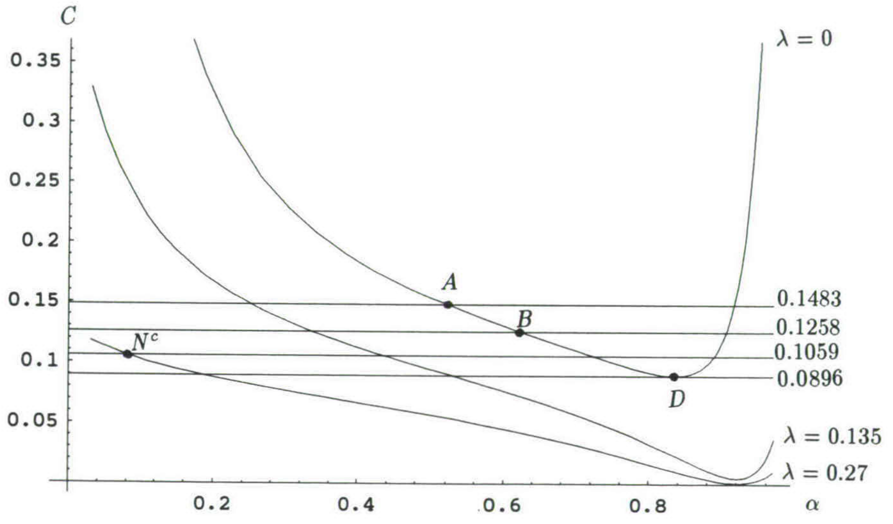


Figure 2.6: For various λ, and varying $\alpha \in [0, 1]$ the corresponding convergence outcome is plotted.

(a) $\gamma < 0.0896$
Both players play a strategy which results in a point on the iso-convergence line I_γ. However (see figure 2.4), for this γ, $I_\gamma \cap P = \emptyset$, and $I_\gamma \cap \mathcal{N} = \emptyset$, which implies that the chosen

strategy is not a Pareto optimal strategy and that the corresponding (J_1, J_2) point falls outside the negotiation area. So, at least one of the players will have higher costs than when he plays the non-cooperative open loop Nash strategy. Such an ambitious setting of the degree of convergence is very unrealistic, it means that convergence prevails over individual costs. Therefore we excluded this possibility in section 2.2 by our assumption of rational behaviour.

(b) $0.0896 \leq \gamma < 0.1059$
For this γ (see figure 2.4), $I_\gamma \cap \mathcal{N} = \emptyset$, but $I_\gamma \cap P \neq \emptyset$. So, a strategy can be played which coincides with the Pareto optimal strategies. From figure 2.6 follows that there are two possible Pareto optimal strategies, which corresponds in both cases wih an $\alpha > 0.625$. So player 2 will have higher costs than when he plays the open loop Nash strategy. On the other hand, playing one of the two Pareto optimal solutions will be very profitable for player 1. Without any other additional agreements between the players (see (a)), player 2 will never accept such an outcome.

(c) $0.1059 \leq \gamma < 0.1258$
In this case, $I_\gamma \cap \mathcal{N} \neq \emptyset$, and $I_\gamma \cap P \neq \emptyset$, but $I_\gamma \cap \mathcal{N} \cap P = \emptyset$. If the players decide to play a Pareto optimal strategy, player 2 will again have higher costs than when playing the open loop Nash strategy. More likely is an outcome within the negotiation area $\mathcal{N}$. For instance if the players agree on a $\gamma = 0.1104$ then they have to find a point in $I_{0.1104} \cap \mathcal{N}$. Looking again at figure 2.4, we see that there is a whole range of possible outcomes. A unique outcome may be obtained using bargaining theory As already can be seen in figure 2.4, any outcome of such a game will be that player 2 does not gain much in a bargaining situation whereas the gains for player 1 can be considerable.

(d) $0.1258 \leq \gamma$
Now, $I_\gamma \cap \mathcal{N} \cap P \neq \emptyset$, which means an outcome can be played on the Pareto line between A, B. Also here, bargaining theory can be applied to select a unique outcome.

Concluding we see that if the desired degree of convergence is set too high the players have to pay a price for that and can not obtain Pareto optimal solutions. Furthermore, if they can not obtain solutions within the negotiation area the player(s) will have an incentive to deviate towards the threatpoint and forget about any degree of convergence at all. Moreover, we observe that in almost all cases player 1 can gain more than player 2. In the figures this depends on the shape of the iso-convergence lines and ultimately is traced back to the fact that player 1 has more influence on player 2 than vice versa.

2.3.3 An approach to determine a reliable degree of convergence

In this section we present an algorithm to determine a feasible degree of γ. The previous subsection states that, without any other agreements between the players, a degree of convergence which has no correspondending outcome in the negotiation area is unlikely to happen. The question remains, however, which degree of convergence within this negotiation area ultimately will be selected by the players. In fact without making any further assumptions on the negotiation process, every point in the negotiation area is possible. One way to come to a unique point within the negotiation area is by axiomatising the negotiation game. We shall not elaborate this subject here, since for the moment we are more interested in qualitative rather than quantitative statements. All we will do is sketch how a feasible degree of γ can be determined, using some heuristic arguments. First we will give an example and then we will present two algorithms which illustrate the approach in general.

In figure 2.7 the convergence value is plotted against the costs of player 1, along the line

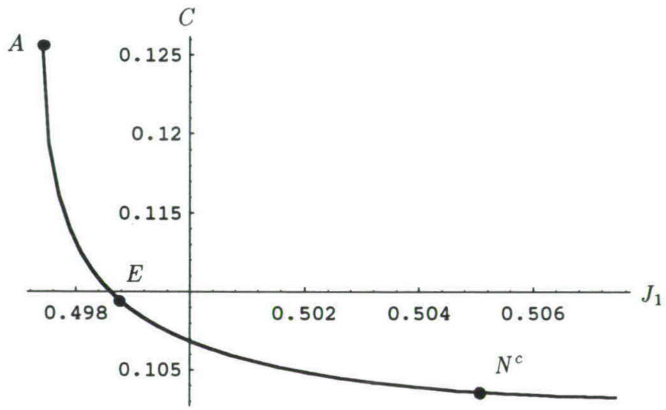


Figure 2.7: For the edge of the negotiation area, from A to N^c, the convergence value is plotted. Point E is the point where the derivative of the tangent of the curve is -1.

A to N^c, where the costs of player 2 remain constant. Starting at point A and moving towards N^c, the convergence value declines rapidly. This continues untill the point where $(J_1, C) = (0.4987, 0.1098)$. After that point the derivative of the slope of the curve gets larger than -1. In figure 2.7 we denoted this point by E. From that point on, towards

N^c, the costs increase more rapidly than the degree of convergence. If player 1 has to choose an outcome on the line in figure 2.7, he will start in point A where his costs are minimal. From thereon, if player 1 wants to increase convergence, he will have to weigh out costs against convergence. For instance, if player 1 starts in A and moves towards N^c and accepts only points where the slope $\partial C/\partial J_1 \leq -1$, the result will be the outcome E.

The general idea expressed in the above example is that players accept an increase in convergence only if the corresponding costs stay within a prespecified region. So, a sketch of a numerical approach for determining a feasible degree of γ would be the following:
(1) Start from a point $(\bar{J}_1, \bar{J}_2)$ on the Pareto line between A, B. It seems reasonable to start at an axiomatic bargaining outcome (see chapter 3).
(2) Determine the direction $v = (v_1, v_2)$, for which there is a $t > 0$ for which $(\bar{J}_1, \bar{J}_2)+t(v_1, v_2) \in N$, and convergence increases maximal.
(3) Choose a small $t > 0$.
(4) Calculate $\chi_c = -\partial C/\partial v$. Check if $\partial J_i/\partial v < \chi_i(\chi_c)$, for $i = 1, 2$ where $\chi_i(\chi_c)$, $i = 1, 2$ are (decreasing) functions of χ_c which indicate the weight players want to assign to the tradeoff between convergence and costs. That is, if the additional increase in convergence (reflected by a smaller value for C) equals χ_c then the additional increase in costs for each player seperately should be less than $\chi_i(\chi_c)$ for $i = 1, 2$.
(5) If (4) holds then use this new point as a starting point and start again in (2). Stop, if no point in $\mathcal{N}$ can be found for which (2) and (4) hold.

A drawback of this approach is that it is rather time-consuming, even for small models. The reason is that the functions J_1, J_2, C are parametrised in α and λ and therefore calculating 'simple looking' expressions like $\partial C/\partial v$ or $\partial J_i/\partial v$ for $i = 1, 2$, or finding a direction v in step (2), take a lot of time.

A good alternative which is strongly related with the previous algorithm, but is easier to compute, is the following algorithm:
(1) start in some feasible point between A, B. With this point there corresponds an uniquely determined α.
(2) Fix α.
(3) Increase λ from 0 to 1 by using a stepsize of, for instance, 0.01. Check if the point stays in the negotiation area $\mathcal{N}$.
(4) Check for every λ whether $-\partial C/\partial\lambda > \partial J_1/\partial\lambda$ and $-\partial C/\partial\lambda > \partial J_2/\partial\lambda$.
(5) Stop if no λ can be found for which (3) and (4) holds.

The conditions in step (4) of the algorithm can be compared with the conditions in step (4) of the previous algorithm. These conditions state that if for each player seperately costs rise less than convergence falls when λ increases by one unit both players are willing to

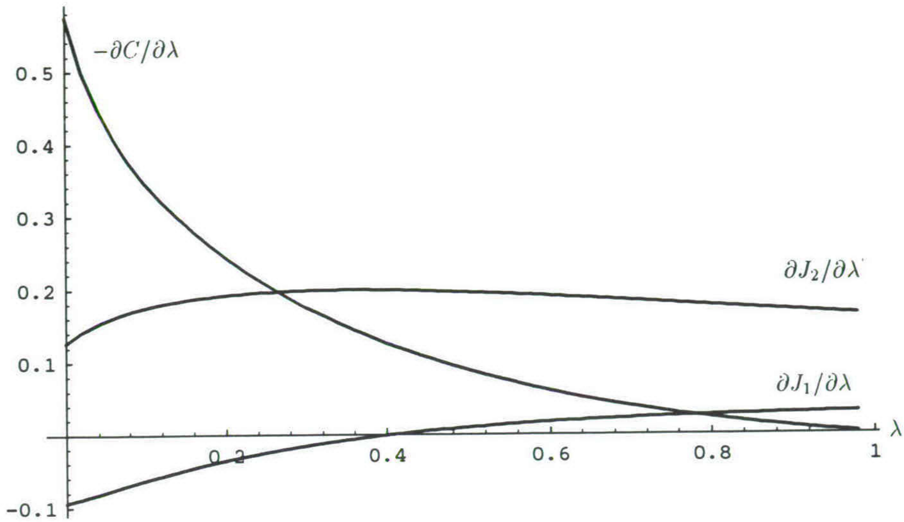


Figure 2.8: For increasing λ, the three curves $-\partial C/\partial\lambda$, $\partial J_1/\partial\lambda$ and $\partial J_2/\partial\lambda$ are drawn.

accept more convergence (as long as they stay within the negotiation area). Note that for our convenience we took $\chi_c = \chi_1(\chi_c) = \chi_2(\chi_c)$. We used the last algorithm to determine a feasible outcome in our example. As a starting point we choose the axiomatic Nash bargaining solution (see chapter 3). This solution corresponds with $\alpha = 0.575$ and lies on the Pareto curve approximatly in the middle between A and B. In figure 2.8 we have drawn for $0 \leq \lambda \leq 1$ the curves $-\partial C/\partial\lambda$, $\partial J_1/\partial\lambda$ and $\partial J_2/\partial\lambda$. The figure shows some interesting facts. First of all the conditions of step (4) of the algorithm are violated when $\lambda > 0.26$. Secondly, when increasing λ the costs of player 1 fall! This lasts till $\lambda = 0.4$ where $\partial J_1/\partial\lambda$ gets positive. On the other hand players' 2 additional costs for increasing convergence are for all λ higher than the additional costs player 1 is faced with. Finally in figure 2.9 we have drawn a small part of the curve:

$$\{(J_1(0.575, \lambda), J_2(0.575, \lambda)) \mid \lambda \in [0, 1]\}$$

The curve tends to stay very close to the Pareto curve and crosses the negotiation area already for a very small $\lambda = 0.04$. This corresponds with a $(J_1, J_2, C) = (0.498, 0.370, 0.1150)$. Remarkable is that player 1 (the stronger player) has lower costs than he would have in the Nash bargaining solution, a solution which would be accepted if the players did not have to reckon with any convergence aspects at all.

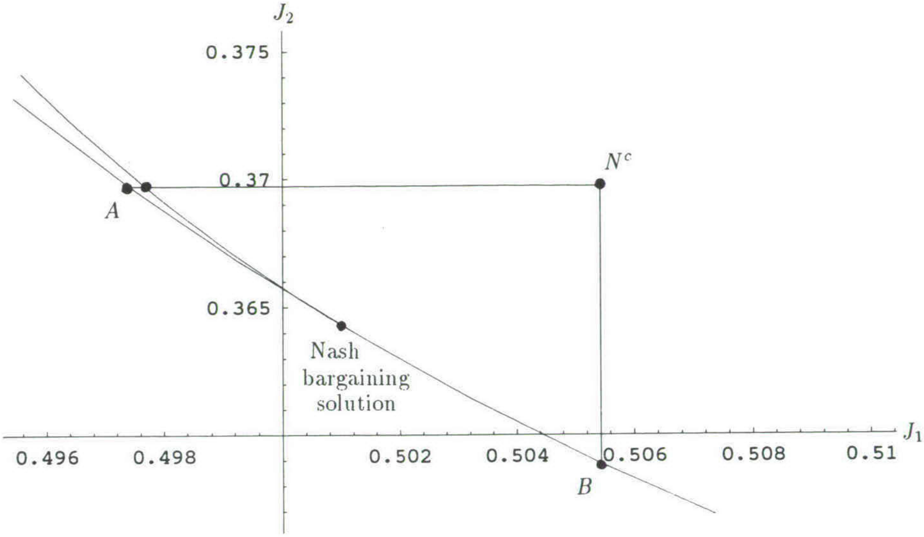


Figure 2.9: Starting from the Nash bargaining solution, fixing α and slowly increasing λ gives a curve which tends to stay very close to the Pareto solutions.

2.4 Conclusions

In this chapter we presented a theoretical approach how to deal with the issue of convergence between EU countries. Based on the assumption that the primary interest of the countries is minimising their own individual welfare loss, we considered the question how cooperative strategies that yield maximal convergence can be determined. We showed that for a large class of problems, i.e. problems where the individual cost functions and convergence function are twice differentiable and convex, a parametrisation for a large set of cooperative strategies can be determined. This set covers the Pareto optimal solutions by construction and, in general (see note in section 2.2), covers all the cooperative strategies which improve over the non-cooperative solution. Using this approach a number of interesting questions can be considered. For instance whether it is possible that for a particular time horizon the EU countries can satisfy the convergence conditions in such a way that for every country the corresponding costs are acceptable, and how these costs differ among countries. In section three we showed in a simple theoretical example how to analyze such questions. In chapter five we use this approach on a more realistic dynamic (macro)econometric model. In dealing with that problem countries should realise that
(1) it must be clear where one should converge to (see van der Ploeg [67]). If we consider the two convergence conditions of the Maastricht Treaty, convergence in the nominal long

term interest rate and convergence in the consumer price deflator, then it is, a priori, not clear what the optimal common rate should be. In our approach this means that countries should agree on a common convergence function C, which contains implicitly the optimal common rate.
(2) the preferences of countries should be finetuned on each other. It is clear that if these preferences differ strongly among countries, convergence will be a very tough issue. In the dynamical game approach this can be analysed with the desired paths and choice of weights for the target/instrumental variables (see Kendrick [52]). The theoretical example was chosen in such a way that in the last period of the planning horizon the countries, at least, strive for convergence, which was implemented by choosing equal values for the corresponding desired paths.
(3) the time-horizon, necessary for reaching the convergence conditions within a limited period, plays a crucial role too. This aspect is strongly related to the determination of the degree of convergence. We expect that for a short planning period the costs for convergence can be very costly and this may ultimately result in non-cooperative behaviour of some countries.This subject remains, however, a topic for future research.
(4) costs for convergence differ among countries. The example in the chapter gives a way how to determine these costs for any given degree of convergence. In general these differences will depend on the economical structures of the participating countries. The theoretical example gives already an indication that these costs could be much higher for countries which have less influence in the Community.

The approach designed here for analysing convergence can be used for many other problems as well. If players in a dynamic game have common objectives, apart from their usual costfunctions, the approach can be used as long as we take twice differentiable convex functions. If we stay in a multicountry setting, common objectives appear in e.g. environmental issues and trade issues.

Chapter 3

N-person axiomatic bargaining solutions

3.1 Introduction

As indicated in the introduction of this thesis we concentrate in this chapter on Pareto optimal policies in the unrestricted cooperative game. We will look at properties of N-person axiomatic bargaining solutions under the technical assumptions that the Pareto frontier is strictly concave and twice differentiable. We study properties of two well known axiomatic bargaining solutions often used in the policy coordination literature, the Nash bargaining and the Kalai-Smorodinsky solution.

Formally, an N-person bargaining problem can be represented by a pair $< S, d >$, where $S \subset \mathbb{R}^N$ is called the feasible set and d the disagreement point or threatpoint. In this chapter we consider a particular class of N-person bargaining problems, i.e., problems where the Pareto frontier of S can be described by a strictly concave and twice differentiable function. Among other areas, this class of problems is extensively studied in the policy coordination literature (see e.g. Ghosh and Masson [34], Hughes Hallett [38, 39], McKibbin and Sachs [55], Oudiz and Sachs [61], Petit [66], Raith [68, 69] and de Zeeuw [85]). In the sequel we will use this stream of literature as a starting point. The class of problems studied in this literature is, however, a subclass of the problems studied in the main stream game theory literature. The main stream game literature considers, generally, problems where $S \subset \mathbb{R}^N$ is a convex set, i.e., the Pareto frontier of S is concave (see, .e.g., Osborne and Rubinstein [60], Peters [65] and Thomson [77]). In the policy coordination literature it is usually assumed that each player maximises his individual utility (or welfare, payoff), where the utility of each player is represented by a strictly concave and twice differentiable function. In that case the Pareto frontier can be found using a maximization problem where a social planner maximises a convex combination of these N utility functions (see, e.g., Takayama [74]). From this also follows that each point on the Pareto frontier is uniquely characterised by a suitable choice of nonnegative weights, say $\alpha_i \geq 0$, $i = 1, ..., N$, which are assigned to the individual utility (or welfare) functions when maximising this convex combination. Without loss of generality one can furthermore assume that $\sum_{i=1}^{N} \alpha_i = 1$. Consequently an outcome on the Pareto frontier of an N-person bargaining problem can be characterised by $N-1$ nonnegative weights. Now, it is common practice in the empirical policy coordination literature to choose some points on the Pareto frontier which can be viewed as acceptable cooperative game outcomes. Since in most (real) policy coordination problems the utility functions of the players are not symmetric, the 'social outcome', which assigns to each player equal weight, is not very representative. It is argued by various authors (see e.g., Ghosh and Masson [34], Petit [66], Raith [68, 69] or de Zeeuw [85]) that the Nash bargaining solution is a more acceptable outcome. In empirical studies of Hughes Hallett [38, 39], Petit [66] and de Zeeuw [85], Nash bargaining outcomes are compared with other axiomatic cooperative approaches, such as the Kalai-Smorodinsky solution [51]. In all the empirical (two player) examples not much differences are found between the corresponding weights of the two most popular cooperative outcomes, the Nash bargaining and the Kalai-Smorodinsky solution. For instance, in de Zeeuw [85] both solutions have the same weights, Petit [66] finds weights which are almost the same, 0.80 and 0.78, and Hughes Hallett [38] reports weights of 0.67 and 0.68. As a result, the authors found also not much differences between the utility function values and corresponding strategy responses of each individual player in both outcomes. To find more variability in the strategy space Hughes Hallett [39] compares the Nash bargaining solution with some other 'arbitrarily chosen weights' outcomes.

Since, till now, the two most 'favourite' axiomatic cooperative outcomes in the policy coordination literature are the Nash bargaining and the Kalai-Smorodinsky outcome, we will restrict ourselves to these two outcomes. Both solutions have been defined for the N-player case, where the Nash bargaining solution can be calculated by maximising a product of the player's benefits in relation to the gains of the disagreement point and the Kalai-Smorodinsky point can be found by determining the intersection point of the line through the disagreement point and the 'ideal point' with the Pareto frontier (see Thomson [77], Nash [59] and Kalai and Smorodinsky [51]).

In the introduction of section two we will formulate the policy coordination problem. Then we will derive in section three for the Nash bargaining solution a unique relationship between the threatpoint, the weights and the utilities the players receive in the Nash bargaining solution. Using this relationship we will present an algorithm for calculating the Nash bargaining outcome which is faster and more reliable than the traditional approach. In the traditional approach the researcher uses a standard optimization algorithm, such as a Gauss-Newton or Gradient method, and maximises a function which is described by the product of the players' benefit in a Pareto optimal point in relation to the gains of the disagreement point. Since, in general, the disagreement point is known and each point on the Pareto curve can be described by a suitable choice of the weights we have that this function depends only on the weights α_i for each player $i, i = 1, ..., N$. In the traditional approach, therefore, the optimization process can be compared with the maximization of a strictly concave function in N variables. In this section, however, we suggest an approach where we first derive a unique relationship for the Nash bargaining solution and next use this relationship to obtain a more computationally efficient algorithm. An other advantage of this relationship is that it enables us to present some strategic arguments for choosing the Nash bargaining solution on the Pareto frontier in policy coordination games (see for similar arguments Ghosh and Masson [34] and Raith [68, 69]). This under the assumption that interpersonal utility is comparable. This approach is quite different from the one used by Osborne and Rubinstein [60], who use an explicit model of bargaining in order to describe the strategic behaviour of the players.

In section four of this chapter we present a possible explanation for the empirical results from the policy coordination literature that the weights corresponding to the Kalai-Smorodinsky and the Nash bargaining solution are almost similar in policy coordination problems. A combination of two arguments makes these findings plausible. First, a theoretical one; we will show that it is possible to design a subset of the Pareto optimal solutions in which both solutions always lie. And second, an empirical one; it seems to be the case that, at least in empirical policy coordination studies, the Pareto curve is often rather flat and, thus, does not contain extreme bendings. Combining the two arguments yields, in general, that the flatter the Pareto curve the closer the two outcomes.

As argued by many authors in the empirical policy coordination literature, one is not so much interested in a certain outcome on the Pareto frontier, but more interested in the properties of these outcomes. One of these properties is how Pareto efficient solutions qualitatively react to small changes of the Pareto curve or the disagreement point. In the policy coordination literature where empirical models are involved this issue is studied by varying the modelparameters. This kind of sensitivity analysis will generally shift the whole Pareto frontier and the question now arises how the Nash bargaining and the Kalai-Smorodinsky outcome react to this parameter change (see Raith [69]). This aspect is interesting since the previous arguments show already that both outcomes often coincide, if we now furthermore could prove that both outcomes react qualitatively in the same manner when applying sensitivity analysis then given our previous computational arguments one could argue that in practice it would be sufficient to calculate only one of the two bargaining outcomes. It is

clear that a theoretical analysis of this subject is difficult, since a good description for the class of the empirical models used in practice is not available, and therefore computing the effect of certain parameter changes is not possible. What we will do in section five of this chapter is that we study the response for the Nash bargaining and Kalai-Smorodinsky solution to certain changes in the disagreement point d, for a fixed Pareto frontier (or, equivalently, a fixed set S). It seems that this type of analyses is closest to the problem sketched above and which is still possible to analyse in a theoretical context. For this analyses, we will follow Thomson [78], who considers this problem from an axiomatic game theoretical point of view. He considers two types of axioms. First, the d-monotonicity axiom. This axiom states, for a fixed set S, that if d_i increases, while for each $j \neq i, d_j$ remains constant then player's i's payoff should increase. Thomson [78] shows that this axiom holds for both solutions if S is a convex subset of $\mathbb{R}^N$. Secondly, he considers the strong d-monotonicity axiom, which states that not only player's i's payoff should increase, but also the payoff's of the other players should decrease. Thomson [78] shows that for the Nash bargaining solution and for the Kalai-Smorodinsky solution this axiom does not generally hold. However, for our special class of bargaining problems, where the Pareto frontier is strictly concave and twice differentiable, we show that this axiom does hold for the Kalai-Smorodinsky solution but not, generally, for the Nash bargaining solution. This finding may, in particular cases, be an argument in favour for the Kalai-Smorodinsky solution since it is clear that in practice, if one is involved in sensitivity analysis with respect to the disagreement point, the strong d-monotonicity property is a useful one.
Another context where these monotonicity properties are relevant are situations in which each player has some control over the position of the disagreement point (see Thomson [78]).

3.2 Problem formulation

In general, a bargaining problem of N-players can be described as $< S, d >$, where $S \subset \mathbb{R}^N$ is compact and convex, $d \in S$, and there exists $J \in S$ such that $J_i > d_i$ for $i = 1, ..., N$ (see, e.g., Osborne and Rubinstein [60]). S is often called the 'feasible set' of utilities. Each element in S represents a tuple $(J_1, ..., J_N)$ where J_i represents the utility (or welfare, payoff) of player $i, i = 1, ..., N$. d is called the 'disagreement point' or 'threatpoint'.
In this chapter we will describe a special case of the bargaining problem which is characteristic for the policy coordination literature. In this literature S is not only a convex set but furthermore it is assumed that the Pareto frontier of S can be described by a strictly concave and twice differentiable function. Formally, the utility (welfare, payoff) for each player $i, i = 1, ..., N$ is assumed to be described by a strictly concave and twice differen-

tiable function J_i in $u \in U$, where U is denoted as the strategy space. Furthermore, we assume that U is a convex and compact set and that each $u = (u_1, ..., u_N) \in U$ contains the strategies of each individual player i, u_i. Now each player wants to maximise utility, i.e., this problem can for each player i be described as:

$$\max_{u \in U} J_i(u), \quad i = 1, ..., N. \tag{3.1}$$

Since each player i has only partly control over u, through u_i, it is clear that in order to solve the maximization problem each player is dependent on the strategy choices of the other player. Now we represent S in the utility space, the $J_1, ..., J_N$ plane, by those outcomes for which each player is individually better out than the utility he would receive in the disagreement point. Thus

$$S = \{J(u) \mid u \in U,\ J(u) = (J_1(u), J_2(u), ..., J_N(u)),\ J(u) \geq d, \}.$$

Remark, first that since $J_i, i = 1, ..., N$ are strictly concave functions, S is a convex set in $\mathbb{R}^N$. And second, since all outcomes in S for each player are better (or at least not worse) than the disagreement point, bargaining on outcomes in S is of interest to all players. The advantage of the above characterisation in (3.1) is that the set of Pareto optimal solutions can be presented formally. First, let U^P be the set of Pareto optimal strategies then this strategy set can be described as: (see, e.g., Takayama [74]).

$$U^P = \{u^* \in U \mid u^* = \arg\max_{u \in U} \sum_{i=1}^{N} \alpha_i J_i(u),\ \alpha_i \geq 0,\ \sum_{i=1}^{N} \alpha_i = 1\} \tag{3.2}$$

In the sequel we assume that U^P lies in the interior of U. This assumption guarantees that u^* is uniquely determined as a function of the parameters $\alpha_1, ..., \alpha_{N-1}$, i.e. $u^* = u(\alpha_1, ..., \alpha_{N-1})$, and that u^* is a continuously differentiable function in $(\alpha_1, ..., \alpha_{N-1})$ (see theorem 2.1). Furthermore, from this characterization we can derive the following property:

Theorem 3.1 *Suppose $J_i(u)$ is strictly concave and twice differentiable in $u \in U$. Let $\alpha_i > 0$ and the corresponding solution in (3.2) be u^*. Let $J_i^* = J_i(u^*)$, for $i = 1, ..., N$,. Then the following holds:*

$$\frac{\partial J_i^*}{\partial J_j} = -\frac{\alpha_j}{\alpha_i}, \tag{3.3}$$

for $i = 1, ..., N, i \neq j$.

Proof. See appendix A.1.

The set of interesting Pareto optimal solutions, $P \subset S$, can be characterised as:

$$P = \{J(u^*) \mid u^* \in U^P,\ J(u^*) \geq d\}$$

P is called the bargaining set (see, e.g., Petit [66]). Remark, that P represents only those outcomes for which all players are better of than in the disagreement point. Thus, in general, P represents only a subset of the Pareto optimal outcomes. Because of the strict concavity and the twice differentiability assumption of $J_i(u)$ in $u \in U$ we have that P represents a hyperplane in the $J_1, ..., J_N$-plane for which it is possible to write $J_i = \varphi(J_1, ..., J_{i-1}, J_{i+1}, ..., J_N)$, for every $i = 1, ..., N$ (see also proof theorem 2.1). Starting from this problem formulation we will study in section three the Nash bargaining solution, which we denote by NB, and in section four and five we compare this solution with the Kalai-Smorodinsky solution, which we denote by KS.

3.3 The Nash bargaining solution

Nash [58] proposed four axiom's on a bargaining solution of the bargaining problem $< S, d >$ which are: (i) Invariance to equivalent utility representations, (ii) Symmetry, (iii) Independence of irrelevant alternatives, and (iv) Pareto efficiency. For a broad discussion about these axiom's see, e.g., Osborne and Rubinstein [60], Peters [65] or Thomson [77]. Nash [58] proved that these four axiom's determine a unique outcome in the utility space which can also be found by considering the following problem:

$$J^{NB} = \arg\max_{J \in P} \prod_{i=1}^{N} (J_i - d_i) \tag{3.4}$$

According to the previous section we have that J^{NB} is determined by exactly one strategy, say u^{NB}, for which $J^{NB} = J^{NB}(u^{NB})$ and that there is also exactly one α, which we will denote by α^{NB}, for which (3.2) yields u^{NB}. In the following subsection we will derive a relationship between d, α^{NB} and J^{NB} which characterise the NB solution in the N-player case. In the next subsection we will use this relationship for deriving an algorithm which computes the NB outcome faster and more reliable than the traditional approach, which is implementing (3.4) straight away. Furthermore, we will discuss in subsection three also some strategic arguments, which make sense in the policy coordination literature.

3.3.1 A relationship between d, α^{NB} and J^{NB}

In this section we will derive a relationship between d, α^{NB} and J^{NB} of the NB solution in the N-player case. For the two player case this relationship is already shown by Nash

[59]. This relationship states that $\alpha_1^{NB}(J_1^{NB} - d_1) = (1 - \alpha_1)^{NB}(J_2^{NB} - d_2)$. In the N-player case this proof yields some additional problems since in that case it is no longer possible to parametrise α by one single element as is usual in the two-player, where $\alpha = (\alpha_1, 1 - \alpha_1)$ depends just on α_1. However, for the N-dimensional case we can derive a similar relationship.
Formally, consider a bargaining problem $< S, d >$ as described in the previous section. Suppose $u = (u_1, ..., u_n)$, where u_i is the strategy of player $i \in 1, .., N$, and $\alpha = (\alpha_1, ..., \alpha_N)$. Then the Pareto optimal solutions P can be derived by solving for each α the maximization problem (3.2) and the unique NB solution can be found by maximising (3.4). This yields the following theorem.

Theorem 3.2 *The following relationship holds between the utilities $J_1^{NB}, ..., J_N^{NB}$ of the players, the threatpoint $d = (d_1, ..., d_N)$ and the weight $\alpha^{NB} = (\alpha_1^{NB}, ..., \alpha_N^{NB})$ of the NB solution:*

$$\alpha_i^{NB} = \frac{\prod_{i \neq j}(J_i^{NB} - d_i)}{\sum_{i=1}^{N} \prod_{i \neq j}(J_i^{NB} - d_i)} \tag{3.5}$$

for $i = 1, ..., N$.

Proof. See Appendix A.2.

Remark that $\alpha > 0$ and that the relationship in theorem 3.2 implies that:

$$\alpha_1^{NB}(J_1^{NB} - d_1) = \alpha_2^{NB}(J_2^{NB} - d_2) = \cdots = \alpha_N^{NB}(J_N^{NB} - d_N), \tag{3.6}$$

To get a better understanding of this result we illustrate in figure 3.1 the proof geometrically for the two-player case (see, e.g., Nash [59], Peters [65] or de Zeeuw [85]). The NB solution on the Pareto curve is the solution for which the angle of the line through $d = (d_1, d_2)$ and (J_1^{NB}, J_2^{NB}) on the Pareto curve and the J_1-axis exactly equals the negative angle of the tangent of the Pareto curve in the point (J_1^{NB}, J_2^{NB}) and the J_1-axis. Both angles are in the figure 3.1 denoted by β. The derivative of the first line is given by $(J_2^{NB} - d_2)/(J_1^{NB} - d_1)$. Now from the figure 3.1 we see that $\tan\beta = (J_2^{NB} - d_2)/(J_1^{NB} - d_1)$. The derivative of the tangent on the Pareto-curve $\alpha J_1 + (1 - \alpha)J_2$ in the NB point is, according to theorem 3.1 given by $\alpha^{NB}/(1 - \alpha^{NB})$. Now from the angle of this slope with the J_1-axis follows that $\tan\beta = (1 - \alpha^{NB})/\alpha^{NB}$. Combining the two outcomes yields that $(1 - \alpha^{NB})(J_2^{NB} - d_2) = \alpha^{NB}(J_1^{NB} - d_1)$.

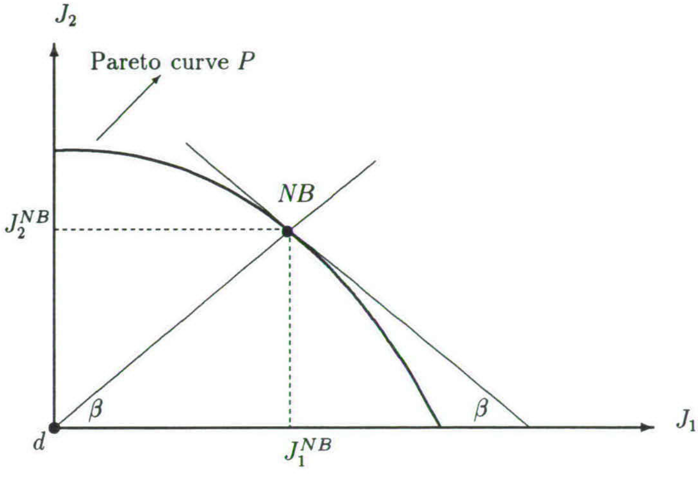


Figure 3.1: A property of the NB solution (NB) in the two-player case.

3.3.2 Numerical calculation

A major advantage of the relationship specified in (3.5) is that numerical calculation in real problems becomes much easier. Before explaining and comparing our algorithm with the traditional approach we will first give a brief description of the traditional approach. Since each point on the Pareto frontier is uniquely determined by a set of $(\alpha_1, ..., \alpha_{N-1})$ we have that in practice the maximization algorithm contains the following steps:
(i) Start with an initial $\alpha^0 = (\alpha_1^0, ..., \alpha_N^0) \in \mathbb{R}^N$, with $\alpha_i^0 \geq 0$, $i = 1, ..., N$ and $\sum_{i=1}^N \alpha_i^0 = 1$. A good guess is often $\alpha^0 = (1/N, ..., 1/N)$.
(ii) Compute (3.2) which yields a Pareto optimal strategy, say $u^* = u(\alpha^0)$.
(iii) Check if $J(u^*) \in P$, if not, use this result for making a new guess for an initial value α^0. Continue this procedure till $J(u(\alpha^0)) \in P$.
(iv) Check whether for this $J(u^*)$, (3.4) holds.
(v) Calculate a new α^1 according to a certain decision rule and compute (ii)-(v).
This algorithm description is typical for problems of finding maximum points of a constrained multivariable function by iterative methods. Most of these algorithms are already implemented in existing computer packages and the type of problems are in the numerical literature generally referred to as constraint non-linear optimization. Since, in many cases, the Pareto frontier can be very flat, the solution of this kind of problem is not straightforward, even if we have a convex surface.

However, the existence of relationship (3.5) leaves us with a non-linear equations problem which facilitates the following approach:
(i)-(iii) as described above.
(iv) Check whether for this $J(u^*)$, (3.5) holds.
(v) as described above.
These type of algorithms are in the numerical literature referred to as non-linear equations problems. There are many solution methods for these kind of problems, such as a Gauss-Newton algorithm or a line-search algorithm (see e.g. Stoer and Bulirsch [73]).
Remark that in a N-player case the outcome in the second algorithm in (iv) already gives an indication which $\alpha_i, i = 1, ..., N$ should be adjusted to a lower value and which one to a higher value.
The main point, however, is that for large problems the second nonlinear equations problems in practice use less computertime than the first constraint maximization problems. This is a well-known fact in the numerical optimization literature. The advantages are clear. Firstly, we do not have to check if the Nash-product really is maximised and secondly step (iv) of the second algorithm will automatically take care of the fact that the α's satisfy the constraints $\alpha_i \geq 0$ and $\sum_{i=1}^{N} \alpha_i = 1$.

3.3.3 Interpretation of the Nash bargaining solution

In this section we present an interpretation of the NB solution which typically fits in the policy coordination literature. In order to make comparisons among the utilities of the players possible we replace in this section Nash's assumption of independence of equivalent utility scaling by the assumption that interpersonal utility is comparable. For a possible interpretation of the NB solution in a more general context we refer to Rubinstein, Safra and Thomson [70]. In the policy coordination literature Ghosh and Masson [34] and Raith [69, 68] describe a possible interpretation of the NB solution in the two player case. Since interpersonal utility is comparable, it is possible to interpret the relationship $\alpha_1^{NB}(J_1^{NB} - d_1) = \alpha_2^{NB}(J_2^{NB} - d_2)$, as that the player who gains more from playing cooperatively is more willing to accept a smaller welfare weight than the player who gains less. Alternatively, the player who gains less may demand a higher welfare weight by threatening not to coordinate, knowing that the potential loss from no agreement is larger for the other player. Using this interpretation we can construct a more general interpretation of the NB solution. For instance in the two player case we assume that each player faces the maximization problem:

$$\max_{u} J_i(u) - d_i \quad \text{for player } i = 1, 2.$$

Now all Pareto efficient strategies of this two-player maximization problem can be found by maximising for every (α_1, α_2) the convex combination:

$$\max_u \alpha_1(J_1(u) - d_1) + \alpha_2(J_2(u) - d_2) \tag{3.7}$$

Now both players simultanously determine α in the following way:
They agree that the more gain a player receives the less weight he will get in the maximization problem. They formalise this agreement by giving player 1 a weight of $(J_2 - d_2)$ and player 2 a weight of $(J_1 - d_1)$, If we substitute these weights in the maximisation problem (3.7), we get:

$$\max_u (J_2(u) - d_2)(J_1(u) - d_1) + (J_1(u) - d_1)(J_2(u) - d_2)$$

which gives us back the original NB problem, which is charactarised by maximising the Nash-product (3.4).
This idea can easily be extended to the N-player case. In that case the weight α_i of player $i = 1, ..., N$, is determined by the product: $(J_1 - d_1) \cdots (J_{i-1} - d_{i-1})(J_{i+1} - d_{i+1}) \cdots (J_N - d_N)$. So, the weight a player gets in the minimisation problem which determines the Pareto optimal strategies is characterised by the product of the gains of the other players.

3.4 A comparison: the Nash bargaining and the Kalai-Smorodinsky solution

Choosing a certain outcome on the Pareto frontier has almost always some arbitrariness. Many objections have been raised against Nash's independent of irrelevant alternatives axiom. To understand this axiom we have to consider a bargaining problem $< S, d >$. Now, if for some reason, the players only have at their disposal a subset of alternatives in S, in which the bargaining outcome of $< S, d >$ is included, then this axioma tells us that the players still agree on the same outcome as in the original bargaining problem. For the two player case an alternative solution is proposed by Kalai and Smorodinsky [51]. They replace Nash's axiom of independence of irrelevant alternatives by an axiom of monotonicity. This axiom requires that if we consider two bargaining problems $< S, d >$ and $< T, d >$, with $S \subset T$, and if the maximum utility a player can obtain in $< S, d >$ and $< T, d >$ are the same then the utility each player receives in $< T, d >$ should be at least as high as in the solution of $< S, d >$. An important feature of the KS solution is that it responds much more satisfactorily to expansions and contractions of the feasible set (see Thomson [77]). The KS solution has mainly been studied for the two-player case, in which it has a greater number of appealing properties than for the N-player case (see Thomson [77]). In practice, the KS solution is computed as follows. Consider a threatpoint $d = (d_1, ..., d_N)$. Compute now N strategies $v_i \in U^P, i = 1, ..., N$ with the resulting property that for each v_i the outcome in S is such that $J_j(v_i) = d_j, j = 1, ..., N$, and $i \neq j$. Remark that these N points are exactly the edge-points of P. These N outcomes determine the so called "ideal point", which can be written as $J^I = (J_1(v_1), ..., J_N(v_N))$. Now the intersection point between the Pareto curve and the line which connects J^I and d yields the KS solution. Remark that to compute the KS outcome one has to solve $N + 1$- non-linear (constraint) equations problems. In practice the computer time involved for computing each of these $N + 1$ non-linear equations problems is about equal to the computer time involved for computing the second algorithm for calculating the Nash bargaining solution, as proposed in the previous section. Therefore, for large problems it takes much more time to compute the KS solution than the NB solution.

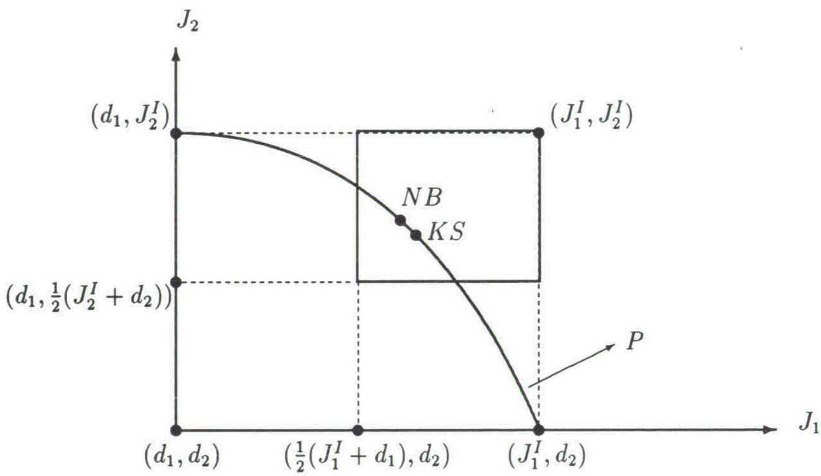


Figure 3.2: The Nash bargaining solution (NB) and the Kalai-Smorodinsky solution (KS) lie on that part of the Pareto curve which intersects the right upper rectangle.

As noted in the introduction the empirical policy coordination literature suggests that both outcomes yield very similar results. These empirical findings have, of course, everything to do with the formulation of the considered bargaining problems. In this section we will prove that it is possible to determine a subset of P that contains both; the NB and the KS solution. Before showing this aspect for the general N-player case, we first take a look at the two-player case. In figure 3.2 we draw the utility axis J_1, J_2 and the curved line which represents P. As can be seen the whole bargaining problem can, in the J_1, J_2-plane, be imbedded in a rectangle with angles (d_1, d_2), (d_1, J_2^I), (J_1^I, d_2) and (J_1^I, J_2^I). This rectangle can be divided in four smaller rectangles of similar shape in exactly one way. Now, as illustrated in the example in the figure 3.2, it will always be the case that the NB solution and the KS solution fall in the upper right rectangle with angles $\frac{1}{2}(J^I + d)$, $(\frac{1}{2}(J_1^I + d_1), J_2^I)$, $(J_1^I, \frac{1}{2}(J_2^I + d_2))$ and J^I. In the following theorem we proof such a property for the general case: the N-person bargaining problem.

Theorem 3.3 *Let $d = (d_1, ..., d_N)$ be the disagreement point and $J^I = (J_1^I, ..., J_N^I)$ be the ideal point. Consider now the N-dimensional cube, say C, with the 2^N angular points:*

$$\{(x_1, ..., x_N) \mid x_i \in \{d_i, J_i^I\}, i = 1, ..., N\}.$$

Let $r_i = d_i + \frac{N-1}{N}(J_i^I - d_i)$, for $i = 1, ..., N$. Now consider the following sets of angular points:

$$\begin{aligned} a_{ii} &= \{(x_1, ..., x_N) \mid \{x_i = d_i \vee x_i = J_i^I\} \wedge x_k = d_k, \quad k = 1, ..., N, k \neq i\}, \\ a_{ij} &= \{(x_1, ..., x_N) \mid \{x_i = d_i \vee x_i = J_i^I\} \wedge x_j = r_j \wedge x_k = d_k, \quad k = 1, ..., N, j \neq k \neq i\}, \\ a_i &= \bigcup_{j=1}^{N} a_{ij}, \end{aligned}$$

for $i, j = 1, ..., N, i \neq j$. Let A_i be the convex polytope of the set a_i. Then both, the NB and the KS solution, will always lie in the truncated cube:

$$C \backslash \{\bigcup_{i=1}^{N} A_i\}$$

Proof. see Appendix A.3.

First, observe that if the Pareto frontier is relatively flat then the Pareto frontier is closer to the $r = (r_1, ..., r_N)$ point (e.g. in the two-player case $r = \frac{1}{2}(d + J^I)$). This implies also that the subset of Pareto optimal outcomes which lie in the truncated cube is small. Secondly, observe that since the KS solution lies on the main diagonal of the truncated cube C we have, in the case of a flat Pareto frontier, that the KS solution will always lie somewhere in the 'centre' of the subset which lies in the truncated cube. Combining both arguments yields that the KS and NB solution will not diverge too much. As the figure 3.2 in the two-player case already suggests, the flatter the Pareto curve, the smaller is the intersection of P with the right upper rectangle and thus the closer are the NB and the KS solution [1]. This aspect seems to be particularly important in the empirical policy

[1] We refer to Appendix A.3.1 for a graphic representation of truncated cube in the three-player case.

coordination literature, since the empirical research in this field suggests that the Pareto curve does not show extreme bendings. For readers who are interested in more examples of figures of Pareto curves in which the KS- and NB-solution are drawn we refer to Hughes Hallett [38], Petit[2] [66] and de Zeeuw [85].

3.5 Strong d-monotonicity properties

Another relevant question, if we are concerned about the choice between the NB and the KS outcome, are the qualitative properties of both solutions. As stated in the introduction of this chapter, in this section we will study the problem how both outcomes, NB and KS change when the disagreement point changes for a fixed set S. There is some research in this field undertaken by Thomson [78] for the more general case where the set S is convex. If, for a fixed set S, the threatpoint d for one particular player, i, increases while for each other player $j, j{\neq}i, d_j$ remains constant, then both solutions recommend an increase in player i's welfare. This property is called d-monotonicity (Thomson [78]). However, Thomson [78] investigates also a stronger requirement, called strong d-monotonicity. This axiom states that if the threatpoint d for one particular player, i, increases while for each other player $j, j{\neq}i, d_j$ remains constant, not only the welfare for player i increases, but also all other players' welfares decrease. Remark, that this property of strong d-monotonicity is always satisfied in two-player bargaining games, since the increase of one player's welfare is always at the costs of the other player's welfare. Less clear are these properties however in the general case: the N-dimensional bargaining game. Thomson [78] shows that in the N-player case (N >3), the stronger requirement that player i is the only one to gain is not generally met. For the 3-player case Thomson [78] gives for both solutions, NB and KS, an example were d_i increases for player i which leads also to an increase in welfare for a player $j, j{\neq}i$, i.e., the general requirement of strong d-monotonicity does not hold for both solutions. For the sequel it is important to remark that both counterexamples were constructed for a bargaining game $< S, d >$ where $S \subset \mathbb{R}^3$ was a convex set in which the surface of S could not be represented by a strictly concave function. Since we are looking here at a smaller class of problems we immediately conclude from Thomson's result that in this case d-monotonicity holds for both outcomes. However, strong d-monotonicity is less clear. In the following theorem we show that the requirement of strong d-monotonicity holds for the Kalai Smorodinsky solution but not, always, for the NB solution.

[2] Remark, that theorem 4.1 implies that the position of the NB-solution and the KS-solution as drawn in figure 9.2 on page 249 cannot be correct.

Theorem 3.4 *Let $< S, d >$ be a N-person bargaining game ($N \geq 3$), where the Pareto frontier of S can be represented by a strictly concave and twice differentiable function. Then the KS solution satisfies the requirement of strong d-monotonicity, whereas the NB solution does not.*

Proof. See appendix A.4.

3.6 Conclusions

In this chapter we considered N-person axiomatic bargaining games for which the Pareto frontier of the feasible set can be described by a strictly concave and twice differentiable function.
For this special class of games we have derived a relationship for the Nash bargaining solution. This relationship describes the Nash bargaining outcome in relation to the threatpoint and the corresponding weights, which follow from maximising a convex combination of individual utility (or welfare) functions. With this relationship, the computation of the Nash bargaining solution becomes far more easier than using the traditional approach, which is maximising the Nash-product straightforward. Since the Nash bargaining solution is commonly used in the policy coordination literature, there is some research in this literature for the strategic reasoning of this solution. If we assume that interpersonal utility is comparable then a possible interpretation might be that the player who gains more by playing cooperatively is more willing to accept a smaller welfare weight. On the other hand, the player who gains less may demand a higher welfare weight by threatening not to coordinate, knowing that the potential loss from no agreement is larger for the other player(s).
The two 'most favourite' solutions used in the the policy coordination literature are the Nash bargaining and the Kalai-Smorodinsky solution. In this chapter we proved for the N-player case that it is possible to derive a subset of the bargaining set, in which both outcomes fall. Combining this result with the empirical results in the policy coordination literature where usually the bargaining set does not show extreme bendings, we have that in practice both solutions mostly lie 'fairly close'. Given the fact that using our algorithm for calculating the Nash bargaining solution is computationally much more efficient than calculating the Kalai-Smorodinsky solution these findings suggests that in policy coordination problems it suffices to calculate the Nash bargaining solution.
In the last section we considered strong d-monotonicity properties, for the N-player case, of both solutions. We investigated how both solutions respond to certain changes in the disagreement point d. If d_i increases, while $d_j, j \neq i$, remains constant, then the Kalai-

Smorodinsky solution recommends an increase of the gains of player i and a decrease in gain for all the other players. This result is opposite to the result which is found for a more general class of games where the feasible set is a convex set. In that class of bargaining games the strong d-monotonicity requirement is not generally met for the Kalai-Smorodinsky solution. Finally we showed that in our class of games, the strong d-monotonicity requirement is, however, not generally met for the Nash bargaining solution. From a policy coordination point of view this suggest that the Kalai-Smorodinsky solution makes more sense. Unfortunately, however, as we noted before the computation of this solution takes more time.

So, our final conclusions for the policy coordination literature are therefore as follows. If the computation of the Nash bargaining and Kalai-Smorodinsky solution are fairly easy to compute, we propose to use the Kalai-Smorodinsky solution as a representative bargaining outcome, since for this solution the property of strong d-monotonicity holds. On the other hand, if the computer-time involved for computing a bargaining outcome is a major problem we suggest to calculate the Nash bargaining outcome since this outcome can, on average, be calculated $N+1$ times quicker than the Kalai-Smorodinsky solution (where N represents the number of players). Furthermore, since in the policy coordination literature the weights corresponding to the Nash bargaining and Kalai-Smorodinsky solution are almost the same, the strategic interpretation we derived for the Nash bargaining solution can also be used for the Kalai-Smorodinsky solution.

Chapter 4

SLIM-A Small Linear Interdependent Model

4.1 Introduction

The aim of this chapter is to build a small linked multi-country model of eight EU-Member States (Belgium, Denmark, France, Germany, Ireland, Italy, the Netherlands and the United Kingdom), the USA and Japan. The model is primarily designed for the dynamic game experiments as described in the previous two chapters. Especially, we want to study empirical gains of coordinated- versus non-coordinated policies if: (1) strong linkages among countries exist, (2) countries differ substantially in size and structure, (3) the number of countries (players) is pretty large. To keep this dynamic game analysis tractable we decided, therefore, to build a small (log-) linear dynamic model in which the main links between countries are modelled. In the applications of dynamic game theory in multi-country models, researchers often use a linearised version of the original non-linear model. The outcomes of the dynamic game experiments on the linearised version should then be representative for the orginal non-linear model (or if not, just holds for its linearised version). For intrinsically non-linear models, the outcome of a dynamic game experiment could be quite different from the outcome, which is obtained from the linearised version of the model. This is another reason why we directly designed a (log-) linear model.

The theoretical starting point will be a modified version of the theoretical Mundell-Fleming model (see, e.g., McKibbin and Sachs [55]). The advantage of this framework is that it is linear and that direct linkages are already in the model. The theoretical model is introduced as an equilibrium model between two countries. We extend this two-country model to more countries using the principal trading pattern of each individual country. As argued by Wallis [80] comparative modelling seems to be the major way to improve macroeconometric models; therefore our model will be compared to the five multi-country models, as mentioned above. We apply the same shocks to our model as Whitley [83] applies to the multi-country models in his comparative study.

The theoretical model contains six linear behavioural equations for each country and will be estimated using annual data for the sample period 1960-1991. The eight EU-Member States represent economies in the European Union for which there is a growing mutual economic activity and for which the annual data, used for estimation, is (almost) completely available. The USA and Japan are included in the model because they are the most important countries outside the EU with the strongest impact on the EU-countries. Due to the increasing integration process between (especially) the EU-countries, external (transmission) effects (by which we mean the effects of how a macroeconomic policy change in one country affects the macroeconomic performance of another country) will become more and more important. In the model we will focus our attention especially on these external effects, which are modelled through direct linkages. The links between the countries considered are of various types. We will include in the model financial links such as interest rates and exchange rates, links between price variables such as consumer prices and GDP-prices and links between volume variables such as output volumes. The economic functioning of the individual economies and their links will be explained in this chapter by carrying out simulation experiments and shock analyses.

There is still a debate in the literature for multi-country studies about the direction (and also strength) of the impact of external effects. In an overview of multi-country modelling Hickman [42] reports of various early multi-country studies where some external effects are investigated. The first research projects on this issue led to the general finding of rather weak spillover effects to other countries from disturbances originating even in large countries. It seems that even today this view has not changed very much. Buiter et all. [13] report the same findings: "Economic multi-country models for simulation imply that under the ERM the output and interest rate effects of a fiscal expansion are confined mostly to the originating EU-country and that the international spillover effects will be insignificant". Furthermore, they quote Bryant et al. [12] who claim that even the sign of a spillover effect is likely to be ambiguous. In a comparative study of five multi-country models, Whitley [83] also finds that in these models spillover effects to the other European countries, originating from single-country European expansion, are negligible. In the case of a fiscal expansion, Whitley reports some quantitative figures of spillover effects: "Spillover effects to the other European countries are largest in MIMOSA (a multi-country model used by the Observatoire Français des Conjonctures Economiques (OFCE) and Centre D'Etudes

Prospectives et D'Informations Internationales (CEPII)), where the increase in GDP in the other countries following a shock in Germany or the United Kingdom is some 20% - 30% of the increase in output in the country shocked; comparable estimates for the other models are around 10%". The other models considered in Whitley [83] are EU's model (QUEST) as operated by the Deutsches Institut für Wirtschaftsforschung (DIW), the model of the National Institute of Economic and Social Research and jointly operated with the London Business School (GEM), the Oxford Economic Forecasting model (OEF) and the OECD's Interlink model (OECD). Whitley also reports figures of the effects on EU-countries of a fiscal expansion originating in the USA. The increase in GDP in the EU-countries following a shock in the USA is on average somewhat higher than 10% of the increase in output in the USA. Again, Whitley finds higher figures which range between 20% and 30% for the MIMOSA model.

We argue, and will show with our model, that these findings are mainly a result of the followed modelling strategies. We will show that with a (slightly) different modelling strategy it is possible to construct stronger external effects. The main reason for finding only small external effects in the large scale multi-country models is that foreign effects are modelled through export or import equations and, so, only indirectly influence aggregate demand. Furthermore, except trade, almost all multi-country models neglect important international transmission mechanisms among countries like foreign investments, labour force migration and various knowledge spill-over effects (see, e.g., Burda and Wyplosz [14] and Whitley [84]).

It is clear that to exactly reveal the international interdependencies is not an easy task. However, the growing interdependence of the EU-economies indicates that the importance of spillovers will further increase in the future which makes it necessary to study models with stronger interdependencies. A priori, e.g., one would expect at least as strong spillover effects in case of an output shock originating in Germany (or another large EU-economy) on other EU-economies as, for instance, in the case of an output shock originating in the USA. There are two basic reasons for this. First, trade among EU-countries is higher than trade with the USA. Second, yearly data show strong correlations between growth output figures of the EU-countries. This last aspect may be due, for a certain part, to similar cyclical behaviour or common shocks but, nevertheless, both aspects indicate that spillover effects among EU-countries could be more important than most multi-country models suggest. Our model is specified such that foreign variables are directly linked with variables of the home country. For example, the aggregate demand equation of the home country will be directly explained by foreign variables, such as foreign output/aggregate demand, the exchange rate and foreign output prices. So, our model does not contain particular export or import categories. This does not necessarily imply that our model is inferior to the large multi-country models, which distinguish many more categories. For instance, in the case of aggregate demand, in our simple model a broad measure such as foreign output/demand represents also indirect effects such as foreign investments and knowledge spill-overs. Such spillovers among countries are better captured by a broad measure such as foreign output/demand than by parts of output such as export or import figures.

The organisation of this chapter is as follows. In section two we introduce the theoretical model. Furthermore, we explain the extension of the two-country model to a multi-country model. In section three we will explain the estimation procedure and our estimation results. Various properties of the model will be investigated in sections four and five. In section four we will present the historical tracking performance of the model. We present static, as well as dynamic, simulation results for all the endogenous variables of the model. In section five we apply shock analysis, and compare the results to five other multi-country models. Finally, we will present our conclusions in section six.

4.2 The theoretical model

As indicated in the Introduction, the starting point of our multi-country model will be a simple two-country model in the Mundell-Fleming framework (see Mundell [57] and Fleming [31]). The main motivation behind this choice is that the Mundell-Fleming framework has a relatively clear economic interpretation, is small, contains direct linkages, and seems to work in practice. A research by Whitley [84, pages 228-231] , who compares the Mundell-Fleming mechanisms with mechanisms as modelled in various multi-country models, states that: 'In summary, therefore, we can conclude that the basic Mundell-Fleming framework and its many extensions can and have been reflected in empirical models, although not all the features are present in many of the models.'. In table 4.1, a modified and extended version of a theoretical two-country Mundell-Fleming model is shown (see, e.g., McKibbin and Sachs [55] for a theoretical interpretation and Papell [63] and Ghosh and Masson [34] for an estimated rational expectations version of the standard Mundell-Fleming model.). This standard Mundell-Fleming model contains an LM-curve which is replaced by a long term interest rate equation in our model, since during the eighties the major monetary policy pursued in the various countries was an interest rate policy. Furthermore, a wage-price spiral is included in this model and, because employment is used as an explanatory variable of wages, we decided to endogenise employment. The model, described in table 4.1, contains five exogenous variables: the exchange rate, government expenditure, labour force, taxes and the short term interest rate. We will use this (almost completely) static model as a starting point for building our multi-country model. In the next subsections we will describe the model and explain how the theoretical model is extended to one which can handle more than two countries.

4.2.1 Description of the theoretical model

The model, as defined in table 4.1, refers to one (home) country. The equations for the second (foreign) country are similar. In the theoretical model, it is assumed that each country produces one (type of) good(s), which is an imperfect substitute for the other country's (type of) good(s). Both (types of) goods are tradable.

The first equation defines goods market equilibrium and is a standard IS-curve for aggregate demand with the real long term interest rate instead of the real short term interest rate. The long term interest rate is interpreted as a measure for the cost of capital. It represents either the costs of borrowing new capital or the opportunity cost of reinvesting retained earnings in the firm. It follows from equation (1) that real aggregate demand is assumed to be a function of the real exchange rate, expressed as $(E + P_y^* - P_y)$, the real long term interest rate, real foreign demand, real government expenditure and real taxes. Important to mention is that P_y is the output price level of the only (type of) good(s) in the home-country and that E is defined as the price of one unit of foreign currency in terms of domestic currency. So, a rise in E corresponds to a depreciation of the home currency. The degree of substitutability between domestic and foreign goods enters this equation explicitly by the Y^*-variable. Theory assumes $\alpha_i \geq 0$ for $i = 1, 2, ..., 5,$. Direct linkages between the domestic and foreign country are modelled in equation (1) through the Y^*- and P_y^*- variables and the exchange rate E.

In equation (2) the output price level is explained in a standard way. The output price level depends on factor costs, which are represented by per capita wages W of the private sector (cost-push inflation). Furthermore, the output price level depends on foreign prices, indicated by $(E + P_y^*)$, and the deviation of gross domestic product from its trend output $\bar{Y}$. All parameters are assumed to be positive.

Consumer prices in equation (3) are assumed to be (positive) linear combinations of domestic output prices P_y and import prices, represented here as $(E + P_y^*)$.

The labour demand function in equation (4) is determined in a fairly standard way. Three factors explain labour demand, namely, real wage costs, output and the gap between foreign and domestic prices. Output is assumed to have a positive effect on labour demand and real wage costs a negative effect. The effect of the difference between the foreign and domestic price levels is ambiguous. From the competitive point of view, if foreign firms raise their prices domestic firms will follow and raise prices as well, in order to get more profits. As a consequence there is room for hiring more labour. From the intermediate product view, a rise in the price of the intermediate (foreign) good will influence the firm's choice between labour and intermediate goods. This choice, however, depends on the production-structure of the firm. In general, we expect a negative sign.

Table 4.1: The theoretical model for one country

Equation number	*Equation*[a]
(1)	$Y = \alpha_1(E + P_y^* - P_y) - \alpha_2(RL - \Delta P_y) + \alpha_3 Y^* + \alpha_4 G - \alpha_5 T$
(2)	$P_y = \gamma_1 W + \gamma_2(E + P_y^*) + \gamma_3(Y - \bar{Y})$
(3)	$P_c = \delta_1 P_y + \delta_2(E + P_y^*)$
(4)	$N = -\eta_1(W - P_y) + \eta_2 Y + \eta_3(E + P_y^* - P_y)$
(5)	$W = \vartheta_1 P_c - \vartheta_2(U - \vartheta_3 U_{-1}) + \vartheta_4(Y - N) - \vartheta 5(P_c - P_y)$
(6)	$RL = \beta_1 RL^* + \beta_2 RS + \beta_3 \Delta(G - T) + \beta_4 \Delta P_c$
(7)	$U = L - N$

a. Variables are defined as follows (asterisks indicate foreign country variables and Δ indicates 'first differences'; all variables, except RS, RL and U which are rates, are in logarithmic form):
Y = real aggregate demand (equal to supply, measured by gross domestic product (GDP))
$\bar{Y}$ = trend volume of real gross domestic product
G = real government expenditure
T = real taxes
RS = nominal short term interest rate
RL = nominal long term interest rate
E = exchange rate defined as the nominal price in domestic currency of a unit of foreign currency
P_y = price level of aggregate demand
P_c = consumer price level
W = nominal wage per employee in the private sector
L = labour force (labour supply)
N = employment
U = unemployment rate

Nominal per capita wages (of the private sector) are modelled in equation (5). They are assumed to depend positively on the consumer prices, according to a price indexing elasticity ϑ_1, and negatively on the unemployment rate, defined as the difference between the (exogenous) labour force and total employment (see definitional equation (7)). With rising unemployment, workers are more solicitous about their jobs as compared to their wages, so their wage claims will be restrained. Moreover, employers will have a larger number of employable workers at their disposal, so their wage offers can be expected to decline. The difference between the short run and the long-run impact of unemployment on the nominal wage is determined by the persistence parameter ϑ_3, which reflects the vulnerability of wages to hysteresis. With prolounged unemployment, the people unemployed can no longer be considered as strong job-candidates who can put significant pressure on the labour market (owing to the deterioration of their skills, motivation and search intensity). In the long-run, the negative effect of unemployment on wages may therefore be much weaker than in the short run. Finally, the nominal per capita wages depend positively on the average labour productivity, represented by $(Y - N)$, assuming that the impact of labour productivity is involved in wage claims, and negatively on the terms of trade, represented by $(P_c - P_y)$ (see, e.g., Heylen [41]) .

Equation (6) explains the domestic long term interest rate. It is assumed that the long term interest rate is explained by the domestic short term interest rate RS, the foreign long-term interest rate RL^*, the changes of government deficit $\Delta(G - T)$ and consumer price inflation which is expressed as ΔP_c. In the sequel we will use the short term interest rate as the price of the money supply and, therefore, it will be used as a policy variable. It is clear, however, that this is a simplification, because, for many countries in a quasi-fixed exchange rate system as the EMS, the short term interest rate is, obviously, a variable which can be controlled only partially. Furthermore, it is assumed that a substantial increase in a country's government deficit will push up the long term interest rate (long term debt financing of the government). By including consumer inflation prices implying a long-run relationship between ongoing inflation and (long term) interest rates, we also consider the Fisher effect.

Due to our precondition to keep the model simple, we had to choose from the outset to take certain effects not into account. For instance, the model excludes expectations, we do not assume that there is a natural rate of unemployment, there are no particular expenditure categories and the model fixes the supply of labour as being exogenous. Furthermore, at this stage, the exchange rate is assumed to be exogenous.

In the next subsection we extend the theoretical model to one which can handle more countries. For this extension we will use the principal trading pattern of each individual country with the other countries considered in the model.

4.2.2 The extension of the theoretical model to more countries

In this section we discuss the extension of the model to one which can handle more than two countries. The structure of each individual country is given in table 4.1. However, we have to specify now how each country's model is linked to the other countries' models. Direct linkages appear in the two-country model through the real exchange rate $(E + P_y^* - P_y)$, the real foreign aggregate demand Y^*, the foreign nominal long term interest rates RL^* and the import prices $(E + P_y^*)$. In macroeconometric modelling, the standard approach for the extension to more than two countries is to consider trade linkages, where the impact of foreign countries is linked through import prices and export and import equations (see, e.g., the Quest model [10]). An other (more simplified) method is estimating the export equations of a home country by adding (trade) weighted averages of foreign outputs (see, e.g., the Taylor model in [76, 75] where, e.g., Y^* in the export equation is replaced by a 'trade weighted average of foreign outputs'). One of the main drawbacks of these approaches is that, in these models, spillover effects among European countries, originating from a single-country European fiscal policy measure, are negligible (see Whitley [83]). A possible reason for these small effects could be that foreign effects are modelled through export or import equations and, so, indirectly influence aggregate demand. Furthermore, using trade weighted averages means that *a priori* the weight of importance of a foreign country is imposed, which can trouble the final outcome. The existence of many more international transmission effects than just trade, makes it likely that the importance of the various international linkages among countries could be different from those suggested by trade figures. The approach we select here is that we do not replace the *-variables in table 4.1 by a trade weighted average of the foreign variables, but incorporate only those countries in the model which were (the most) important trading partners during the sample period 1960-1991. The inclusion of *all* the foreign countries would generate estimation problems because of our limited number of observations. So, instead of *one* $(E + P_y^* - P_y)$- or Y^*- variable in the aggregate demand equation, we got several *-variables, each implying an important foreign country for the domestic country in question. The same approach will be applied for the other equations in table 4.1 which contain foreign variables. As a consequence we get different foreign variables for different countries.

Table 4.2 and table 4.3 give an indication of the importance of trade between the specified countries within the sample period. In table 4.2, the importing share of each country with respect to other countries is given, and in table 4.3, the exporting share of each country. From both tables it is clear that direct linkages exist between (nearly) all countries. Notice that the figures should be read from top to bottom. For instance, the first figure in table 4.2, 2.92, indicates that in the year 1963, 2.92% of Danish total imports came from Belgium. ROW indicates figures for the rest of the world. These figures indicate that the eight EU-countries, USA and Japan are responsible for more than 50% of the importing and exporting shares of each EU-country considered. This is not the case for the exports and imports of the USA and Japan, where the EU-trade shares are below 50%. These facts are, of course, not remarkable because it is well-known that there is a lot of trade within the EU, whereas for the USA and Japan we ignored important trading partners outside the EU.

Table 4.2: Trade Balances: Import percentages for the years 1963, 1977 and 1991

Exporting Country	YEAR	*Importing Country* Belg.	Denm.	Germ.	Fra.	Irel.	Ita.	Nld.	U.K.	Jap.	USA
Belgium	1963		2.92	6.19	7.53	1.91	3.23	17.37	1.88	0.4	2.24
	1977		3.87	8.47	8.98	2.01	3.34	11.84	4.61	0.27	0.98
	1991		3.29	7.97	10.27	2.18	4.88	13.01	4.41	0.67	0.92
Denmark	1963	0.39		2.28	0.62	1.02	1.33	0.7	3.21	0	0.74
	1977	0.46		1.53	0.62	0.66	0.98	0.58	2.23	0.24	0.4
	1991	0.61		2.15	0.91	0.86	0.98	1.13	1.83	0.49	0.34
Germany	1963	19.27	21.04		18.06	6.61	16.9	24.18	5.03	3.27	5.87
	1977	22.25	19.64		18.49	5.91	16.77	24.56	9.82	2.12	4.98
	1991	22.32	23.05		20.64	7.95	21.01	23.65	14.65	4.97	5.62
France	1963	14.95	3.82	10.68		2.41	9.64	5.13	3.44	0.76	2.53
	1977	15.93	4.21	11.67		4.32	13.9	7.02	7.29	0.8	2.08
	1991	14.92	5.87	12.25		4.2	14.15	6.97	9.19	3.26	2.64
Ireland	1963	0.06	0	0.13	0.11		0.09	0.07	2.98	0	0.24
	1977	0.37	0.41	0.39	0.41		0.19	0.49	3.53	0	0
	1991	0.61	0.63	0.81	0.99		0.64	0.76	3.67	0	0.03
Italy	1963	3.5	2.59	7.23	5.94	1.14		2.97	2.3	0.7	2.91
	1977	3.97	3.09	8.85	9.57	2.37		3.48	4.22	0.66	2.08
	1991	4.3	3.98	9.19	11.05	2.23		3.4	5.3	2.17	2.6
Netherl.	1963	14.89	5.52	8.7	4.36	3.3	2.96		4.03	0.74	1.24
	1977	16.77	5.51	13.56	6.1	4.28	4.15		13.98	0.42	1.01
	1991	17.82	7.16	11.78	6.53	5.1	5.76		7.63	0.51	1.02
U.K.	1963	8.33	14.58	4.78	6	51.08	6.18	7.33		2.21	6.29
	1977	7.79	10.94	4.52	5.22	53.31	3.69	6.71		1.3	3.46
	1991	7.96	7.67	6.39	7.58	45.54	5.63	7.94		2.07	4.05
Japan	1963	0.71	0.8	1.03	0.47	1.27	1.24	0.66	1.54		8.87
	1977	1.6	3.11	2.75	1.96	2.05	1.31	2	2.93		12.78
	1991	3.72	3.09	5.3	2.94	3.85	0.28	5.34	5.71		18.05
USA	1963	9.19	8.92	15.47	10.3	5.84	13.64	10.61	11.29	30.85	
	1977	6.03	5.74	6.85	6.94	7.07	6.95	8.53	10.51	17.6	
	1991	5.99	5.78	6.16	8.4	14.87	5.47	7.97	12.6	22.79	
ROW	1963	28.72	39.81	43.52	46.6	25.41	44.78	28.98	64.31	61.07	69.07
	1977	24.83	43.48	41.41	41.7	18	48.73	34.8	40.88	76.6	72.23
	1991	21.77	39.49	37.99	30.7	13.23	41.21	29.83	34.99	63.08	64.72

Table 4.3: Trade Balances: Export percentages for the years 1963, 1977 and 1991

		Exporting Country									
Importing Country	YEAR	Belg.	Denm.	Germ.	France	Irel.	Italy	Netherl.	U.K.	Japan	USA
Belgium	1963		1.12	6.99	9.02	0.59	3.59	14.89	2.81	0.72	2.2
	1977		1.77	7.86	9.96	4.49	3.56	14.66	5.58	1.02	2.59
	1991		2.17	7.33	8.47	4.95	3.42	14.28	5.69	1.35	2.67
Denmark	1963	1.31		3.08	0.98	0	1.04	1.84	2.71	0	0.63
	1977	1.4		2.2	0.81	1.12	0.8	1.71	2.42	0.4	0.44
	1991	0.86		1.87	0.91	0.96	0.79	1.62	1.31	0.34	0.34
Germany	1963	18.56	17.17		16.71	3.36	18.15	26.26	5.3	2.11	4.73
	1977	22.45	15.39		17.1	8.98	18.58	30.87	7.58	3.46	4.98
	1991	23.87	22.52		20.98	12.7	21.25	29.45	13.69	6.2	4.77
France	1963	14.37	3.1	11.12		1.58	10.22	7.74	4.82	0.57	2.91
	1977	19.11	4.32	12.3		7.43	14.29	10.5	6.52	1.25	2.92
	1991	19.04	5.88	13.08		9.45	15.29	10.57	11.07	2.14	3.42
Ireland	1963	0.39	0.43	0.36	0.25		0.17	0.46	3.82	0	0.2
	1977	0.27	0.37	0.28	0.41		0.27	0.42	4.98	0	0
	1991	0.35	0.48	0.44	0.38		0.34	0.61	5.06	0.33	0.65
Italy	1963	5.17	5.19	9.31	9.19	1.38		4.57	3.85	1.49	3.84
	1977	4.38	4.91	6.85	10.47	1.84		4.78	2.97	0.55	2.32
	1991	6.04	4.86	9.12	11.08	4.25		6.32	5.78	1.19	2.05
Netherl.	1963	22.59	2.25	9.34	3.29	0.99	3.63		4.6	1.19	3.22
	1977	16.79	3.51	10.06	5.11	5.69	3.78		6.48	1.62	3.99
	1991	13.67	4.85	8.46	4.67	6.56	3.17		7.9	2.15	3.34
U.K.	1963	5.71	23.26	3.79	4.9	72.53	5.38	9.66		2.85	4.95
	1977	6.85	14.01	5.34	6.5	46.95	5.28	7.47		2.41	4.48
	1991	7.78	10.43	7.67	8.97	32.09	6.67	9.37		3.76	5.85
Japan	1963	0.73	0.48	1.4	0.53	0.2	0.86	0.77	1.31		7.33
	1977	0.45	1.58	1.1	0.72	0.96	0.79	0.53	1.42		8.76
	1991	1.2	3.59	2.47	2.01	2.26	2.21	0.93	2.24		12.47
USA	1963	8.63	6.63	7.2	5.28	6.72	9.49	4.09	8.11	27.97	
	1977	4.19	5.79	6.65	5.14	6.21	6.67	3.43	9.39	24.76	
	1991	3.78	4.9	6.28	6.01	8.68	6.88	3.75	11.08	31.69	
ROW	1963	22.55	40.37	47.4	49.86	12.65	47.46	29.73	62.68	63.11	69.99
	1977	24.12	48.35	47.36	43.77	16.33	45.98	25.64	52.66	64.54	69.52
	1991	23.4	40.33	43.29	36.51	18.1	40	23.11	36.18	50.86	64.46

Table 4.4: Domestic countries and their most important trading partners

Domestic country	most important trading partners
Belgium	Germany, France, Netherlands
Germany	France, Italy, USA, Japan
France	Germany, Italy, United Kingdom, USA
Denmark	Germany, United Kingdom, USA
United Kingdom	Germany, France, USA
Ireland	Germany, United Kingdom, USA
Italy	Germany, France, USA
Netherlands	Belgium, Germany, France, United Kingdom, USA
USA	Germany, Japan
Japan	Germany, USA

Table 4.4 presents our choice of the foreign countries which will be considered as important countries for the domestic country and which will appear as *-variables in the equations of the domestic country. In general, the countries chosen are those with the highest trade share in tables 4.2 and 4.3. However, we must confess that the boundary-lines, determining which country is included as importing/exporting country, are sometimes somewhat arbitrary. For instance, we also took into account that large countries will generate more externalities (e.g., knowledge spillovers) than small countries. Hence, sometimes, a large country was favoured over a small country. For example, we excluded small countries like Belgium and the Netherlands as important trading partners of Germany and France. Furthermore, we included Japan as important trading partner of Germany. Of course, by considering for each country only its most direct linkages many of the existing (weaker) trade linkages between countries are ignored. However, as we will see later on, through the strong direct interaction among countries still (nearly) all countries will be indirectly linked.

Remark that, in our approach, we do not use trade share figures to determine the weight of importance of an effect of a relevant foreign country. In our approach the estimation procedure, which will be explained in the next section, will decide about the weight of importance of a foreign country included. By doing so, we expect to get stronger spillover effects and more differentiation between countries than found in the existing multi-country models. A disadvantage of this estimation procedure can be that multicollinearity may arise between variables of various countries, e.g., owing to similar cyclical behaviour in these countries. Furthermore, spill-overs from countries which are not explicitly modelled are supposedly reflected in the estimated effects of the trade partners which are included in the model.

In the next section we explain our estimation methodology and, finally, present the estimation results for each equation of the model.

4.3 Estimation

We will start this section by explaining the methodology of estimation. Thereafter, we will present in the various subsections the estimation results for each equation separately. For estimation we use yearly data from 1960 till 1991. A description of the data can be found in Appendix B.1.

In general, the equilibrium specification in table 4.1 will be made dynamic according to an Error Correction Mechanism (ECM). In the case of one endogenous variable, e.g., y_t and one explanatory variable, e.g., x_t, the ECM representation relates the current change in y_t to the past deviation of y_t, y_{t-1}, from its long-term path $(\alpha + \beta x_{t-1})$, and to the current change in x_t, as well as to the past changes in x_t and y_t. Such ECM can be written as (see, e.g., Fuss and Sekkat [33]):

$$\begin{aligned} \Delta y_t &= \lambda(y_{t-1} - \alpha - \beta x_{t-1}) + \\ &\quad \delta_0 \Delta x_t + \delta_1 \Delta x_{t-1} + \delta_2 \Delta x_{t-2} + \ldots + \gamma_1 \Delta y_{t-1} + \gamma_2 \Delta y_{t-2} + \ldots + \epsilon_t. \end{aligned} \tag{4.1}$$

In equation (4.1), λ is called the error correction parameter and $(y_{t-1} - \alpha - \beta x_{t-1})$ the error correction term. The speed of adjustment of y_t to its long-term path is determined by λ. It must be negative and less than one in absolute value for the ECM to be stable. In the case that α and β are known, their values can be substituted into equation (4.1), which indentifies the remaining equation. However, α and β are unknown in most cases. Then, in order to estimate equation (4.1) we rewrite the long-run relationship in (4.1) as follows:

$$\lambda(y_{t-1} - \alpha - \beta x_{t-1}) = \lambda_0 + \lambda_1 y_{t-1} + \lambda_2 x_{t-1} \tag{4.2}$$

The approach that we follow here is that we subsitute (4.2) in (4.1) and then estimate equation (4.1) in one step. This unrestricted ECM-procedure is advocated in Banerjee et al. [2] and Inder [47] [1]. In our case, now, the best way to proceed would be to assume that the long-term path for each equation is specified as given in table 4.1. Furthermore, we

[1] In Banerjee et al. [2] and Inder [47] there is some evidence that the estimation of the long-run equation is more efficient and less biased in the unrestricted ECM-procedure than in the two step procedure of Engle and Granger [26], in which the long-term path is estimated first.

would then have to add present (and lagged) changes of each variable in the equilibrium equation of table 4.1 to get an equation as specified in (4.1). However, obviously, if many variables enter the equilibrium equation, the amount of variables which enter the general ECM in (4.1) can expand quickly. In our case we have a very small data set and, therefore, we have to be very careful in selecting the variables for the ECM-model. Since, we use a similar specification across different countries we first choose a long-run relationship wich did reasonably well for all countries, i.e., cointegration seems likely. The next step was to use this long-run relationship in combination with the difference terms and estimate this equation in one step. Therefore, the approach we, finally, decided to use here, and which worked well in practice, is that only domestic variables are chosen to enter the long-run relationship and that all variables of the equilibrium relation, specified in table 4.1, will enter the general equation in difference form. Remark, that with this procedure we give more weight to short-term dynamics than to long-term dynamics.

Note, that with this approach we surpress some of the impact of foreign activity in the long-run. Of course, it seems unlikely that there are no long-run effects between foreign level variables, but we refrained from doing so for practical reasons. For instance, when using the foreign levels of the countries specified in table 4.4, we found for many countries estimation results which yielded bad results for the influence of the level variables of the foreign countries. These results did not disappear when using simultanous estimation procedures. The main reason for these findings is that with the introduction of more simultaneity in the long-run relationships we encounter additional problems such as bias and inconsistency of the estimators in our estimation procedures (see Hendry [40]) and, therefore, stability problems in our dynamic simulation results. With only a limited amount of data available it turned out that these additional problems where very hard to solve, when using an unrestricted ECM-method. Another possibility for modelling the influence of foreign variables, which keeps more control over the weights assigned to the foreign level variables, would be to use constructed weights for the levels of the foreign variables and use these in the long-run relationship. In fact, this approach is close to a restricted form of estimation, which introduces a lot of arbitrary choices such as the determination of the weights. Furthermore, in that case the distribution of the spill-over effects to the foreign countries are fixed beforehand, which was not our original intention. To conclude, with our limited dataset, we could not find reasonable unrestricted estimation results which ultimately yield a model in which the effects of the foreign long-run level variables were modelled properly.

The estimation procedure we use is the *general to specific approach*, in the sense of the Hendry-methodology (see, e.g., Hendry [40]). Considering the limited number of available observations, the general model (4.1) is generally overparameterised. By data-based simplification (i.e., deleting variables with inadmissible parameter impacts and, next, deleting variables with insignificant parameter estimates) the general model is reduced to a more parsimonious model. In the next subsections we specify, for each equation separately, the general dynamic relationship and the decision criteria for determining the simplified equation.

It is common use in macroeconomic modelling that, in order to get a first-shot estimate, each equation is estimated with ordinary least squares (OLS). It is generally known (see, e.g., Brandsma et al. [10]) that simultaneous estimation (e.g., 2SLS, 3SLS) adds little to explanatory power when it is already high in single equation estimations. Furthermore, we have the additional problem that in our case the number of exogenous variables exceeds the number of observations. In the case of simultaneous estimation procedures like 2SLS, 3SLS, we have the additional problem of how to select the appropriate set of instrumental variables. For some cases we used the 3SLS-procedure as a test (for instance in the estimation process of the wage-price spiral). In those cases where 3SLS-results did not correspond (roughly) to the OLS-outcomes, we adopted the approach that we deleted the variable which was responsible for this problem. In most cases that variable could be traced by the fact that it was significant in the OLS-estimation procedure, but insignificant in the 3SLS-estimation procedure. The problem of collinearity is circumvented as follows (see Brunia [11]). If more than one right-hand side variable is found to be significant, then a variable is only retained if it is also significant when the other variable is dropped from the equation, otherwise it is eliminated.

We want to stress that we did not apply the usual tests to see whether variables which enter the long-run relationship are cointegrated or not. There are two reasons for this, which are both related to the fact of a small data set. First, the appropriate tests are asymptotically valid, but the small sample properties of these tests can be questioned (Cochrane [15]). Second, we put much emphasis on getting reasonable static and dynamic simulation results in the process of modelling. If one does so, it can happen that a good fit of a single equation turns out to be a bad equation for the final model.

In this chapter we follow a simple diagnostic which is suggested by Banerjee et al. [2] and use the t-value of λ_1 in (4.1), after substituting (4.2) in (4.1), as a test for cointegration. Now, if this coefficient turned out to be insignificant, which suggests a lack of long-run influence, then we followed the following procedure. We skipped the long-run relationship from the equation if the static and dynamic simulations were worse with this relationship than without it. The estimation results will show that the long-run relationship is fairly significant for all the countries in the GDP-, employment, wage- and long term interest rate equations. This is not the case for the GDP- or consumer price level. This last empirical finding is, however, a common result in macroeconometric modelling. [2].

We are aware of the critique on the general to specific approach that, when applying this methodology, most researchers do not give an exact description of the decisions taken when moving from a general to a simplified model (see, e.g., Pagan [62]). During the process of simplifying the general equation, there is always the interference between decision-making on statistic, economic or simulation grounds and, therefore, a lot of re-estimation has mostly taken place before the final equation is obtained. We should also notice that giving an exact description of the decision-making process of the finally obtained simplified equations for all the sixty (estimated) equations would be very space consuming and, therefore, will not be presented. Hence, we will describe our estimation procedure as clearly as possible, without going into unnecessary details of each estimation before coming up with our final results.

We will present our estimation results for each equation separately. The results are presented with belonging t-statistics, $\bar{R}^2$, the standard error (multiplied by one thousand), SE, and a statistic for (first-order) autocorrelation. Because of the occurrence of lagged dependent variables, the Durbin-Watson statistic is not an appropriate test statistic on first order autocorrelation. Therefore, we used the t-statistic on the estimated autocorrelation coefficient in the following model:

$$\hat{\epsilon}_t = \rho\hat{\epsilon}_{t-1} + \gamma' x_t + \eta_t \tag{4.3}$$

where $\hat{\epsilon}_t, t = 1, ..., T$, are the OLS-residuals from the originally estimated equation: $y_t = \beta' x_t + \epsilon_t$. The statistic $t(\hat{\rho})$ from the OLS-estimation of equation (4.3) is shown for each equation and it should be noticed that the null-hypothesis of zero autocorrelation is accepted if this statistic is smaller than 2 in absolute value. According to Kiviet [53] this autocorrelation test is most useful in the case of small data-sets.

4.3.1 Output

The first equation we consider is the output equation (1) in table 4.1. This equation contains five explanatory variables. Two of these explanatory variables involve foreign variables. As explained in the previous section, each foreign variable has to be replaced by a set of foreign variables as indicated in table 4.4. As already noted, following this approach may increase the number of explanatory variables to a large extent. Since we only have a sample of 32 observations it is clear that we can only consider a subset of

[2]See Whitley [84, page 122] who argues:'Another explanation for the empirical inertia found in price equations is that researchers have found it difficult to find evidence of a co-integrating relationship.'

these variables. As explained in the introduction of this section we do not consider the foreign level variables in the estimation process. Furthermore, we excluded taxes from the estimation process, because it was not possible to find satisfying estimation results for this variable. For most countries, the data of T and G are growing in time with more or less the same speed. Therefore, running a regression which includes both variables did (most of the time) not yield an expected (positive) impact of G and a (negative) impact of T; we also tried the combination $(G - T)$, but this did not work out as well. For three countries, USA, United Kingdom and Ireland, we used the real short term interest rate. According to the literature in the USA, United Kingdom and Ireland, mortgage interest payments are indexed to money market rates. Hence, higher money market rates can impose a significant cost on house-owners (see, e.g., for the United Kingdom and Ireland, Eichengreen and Wyplosz [25], and for the United States, Ghosh and Masson [34]).
Summarizing, we started our approach with the following general aggregate demand model for all countries:

$$\begin{aligned}
\Delta Y_t = \quad & a_0 + a_1 Y_{t-1} + a_2 G_{t-1} + a_3(RL_{t-1} - \Delta P_{y_t}) + \\
& a_4 \Delta G_t + a_5 \Delta(RL_{t-1} - \Delta P_{y_t}) + a_6 \Delta Y_{t-1} + \\
& b_1 \Delta(E_t + P^{*1}_{y_t} - P_{y_t}) + ... + b_k \Delta(E_t + P^{*k}_{y_t} - P_{y_t}) + \\
& c_1 \Delta Y^{*1}_t + ... + c_k \Delta Y^{*k}_t + \\
& a_7 \text{DUM7475} + a_8 \text{DUM7576} + a_9 \text{time} \qquad (4.4)
\end{aligned}$$

For each variable with an asterisk, indicating foreign countries, a foreign country denoted in table 4.4 may appear in the equation. For Belgium, e.g., the general equation implies $k = 3$ because there are three countries, according this table, which may have a significant impact: Germany, France and the Netherlands. As level variables we, finally, included three (domestic) variables, $Y_{t-1}, G_{t-1}, RL_{t-1} - \Delta P_{y_t}$, assuming a long-run relationship between them. Note, that for convenience sake, we do not consider a forward looking term like $\Delta(RL_t - \Delta P_{y_{t+1}})$ [3]. We included step dummies in the general equation: DUM7475 is defined as one for the years 1960-1973 and zero for the years 1974-1991 and DUM7576 is defined accordingly. These dummies belong to the long-run relationship of the general equation and are introduced to capture the oil price shock during the 1973-1974 period. In Perron [64] it is shown that this oil shock had persistent negative effects on domestic GDP growth of oil importing countries. Furthermore, a time dummy is introduced to capture accelerated exogenous growth effects of domestic demand [4]. We expect the sign of $a_2, a_4, a_7, a_8, a_9, b_1, ..., b_k, c_1, ..., c_k \geq 0$, and $a_1, a_3 \leq 0$.

The estimation results are presented in table 4.5. As can be seen from our results, the

[3] In general, the inclusion of this term would not alter the estimation results very much.

[4] Fairly speaking, this time variable should be included in the long-run relationship where it can be interpreted as an exogonous growth component.

level variables $Y_{-1}, G_{-1}, rl_{-1}(= RL_{-1} - \Delta P_y)$ showed significant results in most cases, except in the case of Ireland where G_{-1} did not have an impact according to our supposed theory and, therefore, was excluded. In the case of Japan we could not find any significant impact of the real interest rate. Difference variables of government expenditure appeared in all equations, except Japan and Denmark. In the case of France, Italy and the United Kingdom, the significance of this variable is rather low. Direct linkages are modelled in each equation; however, not all the countries, indicated in table 4.4, yielded significant results. As expected, Germany has a (direct) impact on all other countries except on Ireland. Two countries with considerable influence are also France and the USA; these two countries have direct linkages with five other countries. Especially the impact of GDP growth of France is strong in the equations of Belgium, Germany and the United Kingdom. Real exchange rates effects are largest in the countries Belgium, France and the Netherlands and absent in Denmark, the USA and Japan. A component of foreign growth was found in every country. Large foreign growth effects were found in Belgium, Germany, Denmark, United Kingdom and the Netherlands. In five countries one dummy, capturing the oil shock, had a significant impact and for eight countries the dummy time, indicating exogenous growth, had a significant impact.

4.3.2 The GDP price inflation

Starting from the equilibrium specification in Table 1, we specified ECM-dynamics as indicated in the introduction of this section. It should be stressed that the lagged level component $(Y_{-1} - \bar{Y}_{-1})$, being specified as the demand pull inflation component in table 4.1, had no significant impact in the estimation results and, therefore, will not be considered as level variable in the equilibrium specification of the following general equation:

$$\begin{aligned}\Delta P_{y_t} = \; & a_0 + a_1 P_{y_{t-1}} + a_2 W_{t-1} + \\ & a_3 \Delta P_{y_{t-1}} + a_4 \Delta W_t + a_5 \Delta W_{t-1} + a_6 \Delta(Y - \bar{Y})_t + a_7 \Delta(Y - \bar{Y})_{t-1} + \\ & b_1 \Delta(E + P_y^{*1})_t + ... + b_k \Delta(E + P_y^{*k})_t + \\ & c_1 \Delta(E + P_y^{*1})_{t-1} + ... + c_k \Delta(E + P_y^{*k})_{t-1} \end{aligned} \tag{4.5}$$

For estimating equation (4.5) we followed the same procedure as defined in the previous aggregate demand subsection. Our decision criterium was that all parameteres $a_2, a_4, b_1, ..., b_k, c_1, ...c_k$ should be nonnegative and a_1 should be negative. The signs of a_6 and a_7 are ambiguous, where a positive sign indicates a demand effect and a negative sign a supply effect. This procedure worked quite well for all the ten countries. The results can be found in table 4.6. We have to make some additional remarks. The long-run impact of the level of wages to GDP-prices was restricted to one in Italy. The simulation results

Table 4.5: Estimation results of the aggregate demand equation[a]

Belgium:

$$\begin{array}{ll}
\Delta Y^{Be} = -8.94 - 0.34Y^{Be}_{-1} + 0.14G^{Be}_{-1} - 0.38rl^{Be}_{-1} + 0.29\Delta G^{Be} + 0.17\Delta rl^{Be}_{-1} + 0.17\Delta\lambda^{BeNl} & \overline{R}^2 = 0.80 \\
\qquad (3.45)\ (0.15)\ (0.06)\ (0.16)\ (0.15)\ (0.17)\ (0.08) & SE = 0.09 \\
\qquad 0.13\Delta Y^{Ge} + 0.47\Delta Y^{Fr} + 0.40\Delta Y^{Nl} + 0.026\ \mathrm{DUM7576} + 0.006 time & t(\hat{\rho}) = -0.97 \\
\qquad (0.15)\ (0.23)\ (0.18)\ (0.01)\ (0.002) &
\end{array}$$

Germany:

$$\begin{array}{ll}
\Delta Y^{Ge} = -6.30 - 0.37Y^{Ge}_{-1} + 0.22G^{Ge}_{-1} - 0.33rl^{Ge}_{-1} + 0.51\Delta G^{Ge} - 0.42\Delta rl^{Ge}_{-1} + 0.26\Delta Y^{Us} & \overline{R}^2 = 0.82 \\
\qquad (2.16)\ (0.14)\ (0.08)\ (0.21)\ (0.10)\ (0.19)\ (0.10) & SE = 0.08 \\
\qquad +0.52\Delta Y^{Fr} + 0.09\Delta Y^{Ja} + 0.03\Delta\lambda^{GeUs} + 0.031\ \mathrm{DUM7475} + 0.004 time & t(\hat{\rho}) = -0.32 \\
\qquad (0.19)\ (0.12)\ (0.02)\ (0.015)\ (0.002) &
\end{array}$$

France:

$$\begin{array}{ll}
\Delta Y^{Fr} = -5.16 - 0.11Y^{Fr}_{-1} + 0.03G^{Fr}_{-1} - 0.33rl^{Fr}_{-1} + 0.12\Delta G^{Fr} + 0.28\Delta Y^{Uk} + 0.16\Delta Y^{Ge} & \overline{R}^2 = 0.82 \\
\qquad (2.53)\ (0.06)\ (0.06)\ (0.14)\ (0.14)\ (0.11)\ (0.10) & SE = 0.06 \\
\qquad +0.10\Delta\lambda^{FrGe} + 0.06\Delta\lambda^{FrUk} + 0.023\ \mathrm{DUM7475} + 0.003 time & t(\hat{\rho}) = -0.73 \\
\qquad (0.04)\ (0.03)\ (0.010)\ (0.002) &
\end{array}$$

Denmark:

$$\begin{array}{ll}
\Delta Y^{Dn} = -2.39 - 0.43Y^{Dn}_{-1} + 0.14G^{Dn}_{-1} - 0.13rl^{Dn}_{-1} + 0.46\Delta Y^{Ge} + 0.28\Delta Y^{Uk} & \overline{R}^2 = 0.82 \\
\qquad (2.22)\ (0.10)\ (0.04)\ (0.11)\ (0.12)\ (0.11) & SE = 0.12 \\
\qquad +0.003\ time & t(\hat{\rho}) = -0.80 \\
\qquad (0.001) &
\end{array}$$

United Kingdom:

$$\begin{array}{ll}
\Delta Y^{Uk} = -12.6 - 0.32Y^{Uk}_{-1} + 0.04G^{Uk}_{-1} - 0.17rs^{Uk}_{-1} + 0.05\Delta G^{Uk} - 0.08\Delta rs^{Uk}_{-1} + 0.09\Delta\lambda^{UkGe} & \overline{R}^2 = 0.68 \\
\qquad (5.61)\ (0.13)\ (0.05)\ (0.13)\ (0.07)\ (0.08)\ (0.04) & SE = 0.15 \\
\qquad +0.61\Delta Y^{Fr} + 0.36\Delta Y^{Us} + 0.02\ \mathrm{DUM7475} + 0.008 time & t(\hat{\rho}) = -0.34 \\
\qquad (0.24)\ (0.36)\ (0.02)\ (0.004) &
\end{array}$$

Ireland:

$$\begin{array}{ll}
\Delta Y^{Ir} = -48.9 - 0.66Y^{Ir}_{-1} - 0.38rs^{Ir}_{-1} + 0.21\Delta G^{Ir} + 0.15\Delta rs^{Ir}_{-1} + 0.43\Delta Y^{Ir}_{-1} + 0.22\Delta Y^{Us} & \overline{R}^2 = 0.35 \\
\qquad (11.8)\ (0.16)\ (0.13)\ (0.10)\ (0.11)\ (0.18)\ (0.21) & SE = 0.32 \\
\qquad +0.03\Delta\lambda^{IrUs} + 0.07\Delta\lambda^{IrUk} + 0.028\ time & t(\hat{\rho}) = 0.35 \\
\qquad (0.04)\ (0.07)\ (0.007) &
\end{array}$$

Italy:

$$\begin{array}{ll}
\Delta Y^{It} = 0.64 - 0.11Y^{It}_{-1} + 0.06G^{It}_{-1} + 0.07\Delta G^{It} - 0.45\Delta rl^{It}_{-1} + 0.34\Delta Y^{Fr} + 0.06\Delta Y^{Us} & \overline{R}^2 = 0.46 \\
\qquad (0.49)\ (0.08)\ (0.05)\ (0.08)\ (0.08)\ (0.22)\ (0.11) & SE = 0.27 \\
\qquad +0.02\Delta\lambda^{ItUs} + 0.07\Delta\lambda^{GeIt} & t(\hat{\rho}) = 0.59 \\
\qquad (0.02)\ (0.07) &
\end{array}$$

Netherlands:

$$\begin{array}{ll}
\Delta Y^{Nl} = -3.00 - 0.20Y^{Nl}_{-1} + 0.10G^{Nl}_{-1} - 0.31rl^{Nl} + 0.35\Delta G^{Nl} + 0.25\Delta rl^{Nl}_{-1} + 0.25\Delta Y^{Ge} & \overline{R}^2 = 0.83 \\
\qquad (2.41)\ (0.11)\ (0.06)\ (0.24)\ (0.13)\ (0.13)\ (0.13) & SE = 0.08 \\
\quad +0.27\Delta Y^{Fr} + 0.46\Delta Y^{Be} + 0.21\Delta\lambda^{NlBe} + 0.13\Delta\lambda^{NlGe} + 0.02\Delta\lambda^{NlUs} + 0.002\ time & t(\hat{\rho}) = -0.57 \\
\quad (0.26)\ (0.18)\ (0.07)\ (0.12)\ (0.02)\ (0.002) &
\end{array}$$

USA:

$$\begin{array}{ll}
\Delta Y^{Us} = -7.36 - 0.50Y^{Us}_{-1} + 0.21G^{Us}_{-1} - 0.26rs^{Us}_{-1} + 0.31\Delta G^{Us} - 0.38\Delta rs^{Us}_{-1} + 0.39\Delta Y^{Us}_{-1} & \overline{R}^2 = 0.64 \\
\qquad (5.11)\ (0.18)\ (0.12)\ (0.21)\ (0.23)\ (0.24)\ (0.15) & SE = 0.19 \\
\qquad +0.33\Delta Y^{Ge} + 0.006\ time & t(\hat{\rho}) = 2.03 \\
\qquad (0.17)\ (0.003) &
\end{array}$$

Japan:

$$\begin{array}{ll}
\Delta Y^{Ja} = 0.52 - 0.12Y^{Ja}_{-1} + 0.09G^{Ja}_{-1} + 0.42\Delta Y^{Ge} + 0.06\ \mathrm{DUM7475} & \overline{R}^2 = 0.73 \\
\qquad (0.22)\ (0.05)\ (0.04)\ (0.17)\ (0.02) & SE = 0.29 \\
 & t(\hat{\rho}) = 0.79
\end{array}$$

a. *The real echange rate between two countries, e.g., Belgium and the Netherlands in the first equation,* λ^{BeNl}, *is defined as* $E + P^{Nl} - P^{Be}$, *where* E *is the exchange rate between Belgium and the Netherlands, defined as the amount of Belgian Francs for one Dutch Guilder. The real long term interest rate is defined as* $rl_{-1} := RL_{-1} - \Delta P_y$ *and the real short term interest rate as* $rs_{-1} := RS_{-1} - \Delta P_y$.

Table 4.6: Estimation results of the GDP-price equation[a]

Country	Equation	$\bar{R}^2$	SE	$t(\hat{\rho})$
Belgium:	$\Delta P_y^{Be} = \underset{(0.62)}{-0.36} + \underset{(0.13)}{0.31}\Delta P_{y-1}^{Be} + \underset{(0.11)}{0.29}\Delta W^{Be} + \underset{(0.13)}{0.32}\Delta(Y-\bar{Y})_{-1}^{Be} + \underset{(0.07)}{0.18}\Delta P^{BeNl} \underset{(0.075)}{-0.036} P_{y-1}^{Be} + \underset{(0.044)}{0.026} W_{-1}^{Be}$	0.82	0.11	1.48
Germany:	$\Delta P_y^{Ge} = \underset{(0.28)}{-0.28} + \underset{(0.10)}{0.27}\Delta P_{y-1}^{Ge} + \underset{(0.08)}{0.45}\Delta W^{Ge} - \underset{(0.08)}{0.18}\Delta(Y-\bar{Y})^{Ge} + \underset{(0.07)}{0.15}\Delta(Y-\bar{Y})_{-1}^{Ge} + \underset{(0.011)}{0.014}\Delta P_{-1}^{GeUs} - \underset{(0.045)}{0.028} P_{y-1}^{Ge} + \underset{(0.044)}{0.026} W_{-1}^{Ge}$	0.87	0.03	-1.07
France:	$\Delta P_y^{Fr} = \underset{(0.36)}{-0.66} + \underset{(0.07)}{0.29}\Delta P_{y-1}^{Fr} + \underset{(0.06)}{0.57}\Delta W^{Fr} - \underset{(0.10)}{0.13}\Delta(Y-\bar{Y})_{-1}^{Fr} + \underset{(0.01)}{0.02}\Delta P_{-1}^{FrUs} - \underset{(0.027)}{0.037} P_{y-1}^{Fr} + \underset{(0.019)}{0.035} W_{-1}^{Fr}$	0.97	0.03	-0.29
Denmark:	$\Delta P_y^{Dn} = \underset{(0.55)}{-1.22} + \underset{(0.14)}{0.33}\Delta P_{y-1}^{Dn} + \underset{(0.11)}{0.39}\Delta W^{Dn} - \underset{(0.08)}{0.25}\Delta(Y-\bar{Y})^{Dn} + \underset{(0.09)}{0.16}\Delta(Y-\bar{Y})_{-1}^{Dn} + \underset{(0.03)}{0.03}\Delta P_{-1}^{DnUk} + \underset{(0.02)}{0.02}\Delta P_{-1}^{DnUs} - \underset{(0.057)}{0.126} P_{y-1}^{Dn} + \underset{(0.044)}{0.101} W_{-1}^{Dn}$	0.88	0.08	-0.03
United Kingdom:	$\Delta P_y^{Uk} = \underset{(0.64)}{-0.80} + \underset{(0.09)}{0.71}\Delta P_{y-1}^{Uk} - \underset{(0.16)}{0.56}\Delta(Y-\bar{Y})^{Uk} + \underset{(0.23)}{0.99}\Delta(Y-\bar{Y})_{-1}^{Uk} + \underset{(0.02)}{0.14}\mathrm{DUM75} - \underset{(0.08)}{0.11} P_{y-1}^{Uk} + \underset{(0.08)}{0.08} W_{-1}^{Uk}$	0.89	0.26	0.18
Ireland:	$\Delta P_y^{Ir} = \underset{(0.69)}{-0.44} + \underset{(0.11)}{0.67}\Delta W^{Ir} - \underset{(0.16)}{0.43}\Delta(Y-\bar{Y})^{Ir} + \underset{(0.06)}{0.25}\Delta P_{-1}^{IrUk} - \underset{(0.098)}{0.058} P_{y-1}^{Ir} + \underset{(0.072)}{0.044} W_{-1}^{Ir}$	0.89	0.33	-0.38
Italy:	$\Delta P_y^{It} = \underset{(0.03)}{-0.13} + \underset{(0.08)}{0.40}\Delta P_{y-1}^{It} + \underset{(0.08)}{0.55}\Delta W^{It} + \underset{(0.12)}{0.22}\Delta(Y-\bar{Y})^{It} + \underset{(0.05)}{0.06}\Delta P^{ItGe} - \underset{(0.011)}{0.035}(P_{y-1}^{It} - W_{-1}^{It})$	0.95	0.13	-0.19
Netherlands:	$\Delta P_y^{Nl} = \underset{(0.37)}{-0.56} + \underset{(0.07)}{0.57}\Delta W^{Nl} - \underset{(0.08)}{0.17}\Delta(Y-\bar{Y})^{Nl} + \underset{(0.06)}{0.18}\Delta P_{-1}^{NlBe} + \underset{(0.02)}{0.07}\Delta P^{NlUs} - \underset{(0.054)}{0.064} P_{y-1}^{Nl} + \underset{(0.051)}{0.051} W_{-1}^{Nl}$	0.94	0.05	-0.20
USA:	$\Delta P_y^{Us} = \underset{(1.30)}{-3.72} + \underset{(0.12)}{0.71}\Delta P_{y-1}^{Us} + \underset{(0.10)}{0.11}\Delta(Y-\bar{Y})_{-1}^{Us} - \underset{(0.08)}{0.24} P_{y-1}^{Us} + \underset{(0.08)}{0.22} W_{-1}^{Us}$	0.80	0.10	0.58
Japan:	$\Delta P_y^{Ja} = \underset{(0.82)}{-1.51} + \underset{(0.16)}{0.57}\Delta P_{y-1}^{Ja} + \underset{(0.13)}{0.74}\Delta W^{Ja} - \underset{(0.16)}{0.54}\Delta W^{Ja} - \underset{(0.18)}{0.30}\Delta(Y-\bar{Y})^{Ja} + \underset{(0.14)}{0.14}\Delta(Y-\bar{Y})_{-1}^{Ja} - \underset{(0.10)}{0.17} P_{y-1}^{Ja} + \underset{(0.05)}{0.10} W_{-1}^{Ja}$	0.83	0.22	-1.21

a. The competetive GDP price between a home and a foreign country, e.g., Belgium and the Netherlands in the first equation, P^{BeNl}, is defined as $E + P_y^{Nl}$, where E is the exchange rate between Belgium and the Netherlands, defined as the amount of Belgian Francs for one Dutch Guilder.

improved considerably when imposing this restriction. Furthermore, a first difference wage effect, which was significant in the original OLS-regression but not in the 3SLS-estimation, was excluded in the USA and in the United Kingdom. In the case of the GDP-price equation of the United Kingdom, the impact of the GDP-price of the year 1975 worked as a lever. Therefore we included a dummy DUM75, which is one in 1975 and zero elsewhere, into the equation.

The multipliers of differences in short run per capita wage costs (ΔW) are in the same range (0-0.67) as published in the Quest model[10]. However, there are some differences among countries, which can be explained, not only by the different data samples, but also by the different sets of variables which were taken into account during the estimation process. In our case, we included foreign variables in the estimation process and this had a serious effect in almost every country as can be seen from the results. As expected small countries, such as Belgium, the Netherlands and Ireland have strong foreign price effects. In the Netherlands we see the remarkable fact that the lagged (home) inflation variable disappeared but, instead, two foreign inflation variables were included. The appearence of these foreign variables indicate a high degree of openness for the economy in the Netherlands. We see that the USA have a strong impact on other countries. Its competitive GDP inflation level had substantial effects in four of the eight EU-countries. The variable $\Delta(Y - \bar{Y})$ had serious effects in all the countries. In general, except for Italy and France, the variable had a positive lagged effect and a negative current effect, indicating a cyclical price behaviour. A rise in output instantanously lowers prices, and raises prices one year later. Note, that the sign of the overall effect of a change in output from trend is unclear.

4.3.3 The consumer price inflation

The equation for the consumer price inflation was estimated in the same way as the GDP price inflation. We used the equilibrium equation from table 4.1 and we made a dynamic formulation of it as explained in the introduction of this section. A long-run relationship in each country was assumed between the consumer price level P_c and the output price level P_y. The general equation is specified as follows:

$$\begin{aligned}\Delta P_{c_t} = \quad & a_0 + a_1 P_{c_{t-1}} + a_2 P_{y_{t-1}} + \\ & a_3 \Delta P_{y_t} + a_4 \Delta P_{y_{t-1}} + a_5 \Delta P_{c_{t-1}} \\ & b_1 \Delta(E + P^{*1}_{y_t}) + ... + b_k \Delta(E + P^{*k}_{y_t}) + \\ & c_1 \Delta(E + P^{*1}_{y_t-1}) + ... + b_k \Delta(E + P^{*k}_{y_{t-1}})\end{aligned} \tag{4.6}$$

The estimation results, after applying the general to specific estimation scheme, can be found in table 4.7. Our prior belief was that all parameters should be nonnegative, except

a_1 which should be negative. We have to remark that the error correction term (indicated by the lagged level variables) was excluded for Belgium, Ireland, Italy, Netherlands and USA, since the significance of these terms was very low and the simulation results turned out to be better without these terms. However, consumer price levels and output price levels are still very strongly related with each other in all countries. The reason for this is that the most important indicator for the consumer price inflation is the GDP price inflator. If we look at the home effects indicated by $\Delta P_{y_t}, \Delta P_{y_{t-1}}$ and $\Delta P_{c_{t-1}}$ then we see that in almost all cases these variables explain more than 80 % of the consumer price inflation. We see that Germany had a significant effect in all countries, except in France and Denmark. If we consider the multipliers of the foreign effects, we see that small countries like Belgium, Denmark, Ireland and the Netherlands are mostly influenced by foreign countries. This is not astonishing because it is well known that these economies are the most open ones of all the EU-countries[5]. The long-run elasticity, of the long-run relationship between the consumer price and the GDP-price, equals almost one in those cases where it had a significant impact.

4.3.4 The employment equation

For the general specification of total employment in the individual economies we followed the scheme which includes all home level variables of the equilibrium equation, as specified in table 4.1, and all present and past changes of all the variables. Furthermore, we included a time dummy which represents an autonomous (technology) trend. The general equation is specified as follows:

$$\begin{aligned}\Delta N_t = \quad & a_0 + a_1 N_{t-1} + a_2(W - P_y)_{t-1} + a_3 Y_{t-1} \\ & + a_4 \Delta N_{t-1} + a_5 \Delta(W_t - P_{y_t}) + a_6 \Delta(W_{t-1} - P_{y_{t-1}}) + \\ & a_7 \Delta Y_t + a_8 \Delta Y_{t-1} + a_9 \text{time} + \\ & b_1 \Delta(E_t + P^{*1}_{y_t} - P_{y_t}) + \ldots + b_k \Delta(E_t + P^{*k}_{y_t} - P_{y_t}) + \\ & c_1 \Delta(E_{t-1} + P^{*1}_{y_{t-1}} - P_{y_{t-1}}) + \ldots + c_k \Delta(E_{t-1} + P^{*k}_{y_{t-1}} - P_{y_{t-1}}) \end{aligned} \tag{4.7}$$

According to economic theory our priors were that $a_1, a_2, a_5 \leq 0$ and $a_3, a_7 \geq 0$. The estimation results can be found in table 4.8. The error-correction parameter was negative and in absolute value smaller than one in all cases, except for Ireland were it did not occur, indicating a stable relationship. Furthermore, most variables in the error correction term (determined by the level variables $N_{t-1}, W_{r_{t-1}} (:= W_{t-1} - P_{y_{t-1}}), Y_{t-1}$) proved to be significant. However, in some cases the level effect of real wages disappeared. Notice that

[5]For measures of openness, see, e.g., the Quest model [10]

Table 4.7: Estimation results of the consumer price equation[a]

Belgium:

$$\Delta P_c^{Be} = \underset{(0.004)}{-0.008} + \underset{(0.09)}{0.81}\Delta P_y^{Be} + \underset{(0.05)}{0.07}\Delta P_{-1}^{BeGe} + \underset{(0.05)}{0.13}\Delta P^{BeFr} + \underset{(0.05)}{0.12}\Delta P_{-1}^{BeFr} \qquad \overline{R}^2 = 0.87, \; SE = 0.10, \; t(\hat{\rho}) = -0.35$$

Germany:

$$\Delta P_c^{Ge} = \underset{(0.004)}{0.007} + \underset{(0.10)}{0.40}\Delta P_y^{Ge} + \underset{(0.08)}{0.32}\Delta P_{c_{-1}}^{Ge} + \underset{(0.01)}{0.03}\Delta P^{GeFr} + \underset{(0.01)}{0.06}\Delta P^{GeUs} \underset{(0.05)}{-0.24}P_{c_{-1}}^{Ge} + \underset{(0.05)}{0.22}P_{y_{-1}}^{Ge} \qquad \overline{R}^2 = 0.91, \; SE = 0.03, \; t(\hat{\rho}) = 1.32$$

France:

$$\Delta P_c^{Fr} = \underset{(0.00)}{0.00} + \underset{(0.05)}{0.94}\Delta P_y^{Fr} + \underset{(0.02)}{0.03}\Delta P^{FrUk} + \underset{(0.01)}{0.04}\Delta P^{FrUs} \underset{(0.12)}{-0.37}P_{c_{-1}}^{Fr} + \underset{(0.13)}{0.37}P_{y_{-1}}^{Fr} \qquad \overline{R}^2 = 0.96, \; SE = 0.04, \; t(\hat{\rho}) = -0.32$$

Denmark:

$$\Delta P_c^{Dn} = \underset{(0.01)}{-0.00} + \underset{(0.14)}{0.81}\Delta P_y^{Dn} + \underset{(0.14)}{0.15}\Delta P_{y_{-1}}^{Dn} + \underset{(0.03)}{0.06}\Delta P^{DnUk} \underset{(0.09)}{-0.18}P_{c_{-1}}^{Dn} + \underset{(0.09)}{0.18}P_{y_{-1}}^{Dn} \qquad \overline{R}^2 = 0.85, \; SE = 0.13, \; t(\hat{\rho}) = -0.32$$

United Kingdom:

$$\Delta P_c^{Uk} = \underset{(0.00)}{0.00} + \underset{(0.03)}{0.89}\Delta P_y^{Uk} + \underset{(0.02)}{0.02}\Delta P^{UkGe} + \underset{(0.02)}{0.07}\Delta P_{-1}^{UkFr} + \underset{(0.02)}{0.03}\Delta P^{UkUs} \underset{(0.09)}{-0.47}P_{c_{-1}}^{Uk} + \underset{(0.09)}{0.45}P_{y_{-1}}^{Uk} \qquad \overline{R}^2 = 0.98, \; SE = 0.05, \; t(\hat{\rho}) = -1.37$$

Ireland:

$$\Delta P_c^{Ir} = \underset{(0.006)}{-0.004} + \underset{(0.11)}{0.43}\Delta P_y^{Ir} + \underset{(0.09)}{0.13}\Delta P_{y_{-1}}^{Ir} + \underset{(0.07)}{0.09}\Delta P^{IrGe} + \underset{(0.07)}{0.33}\Delta P^{IrUk} + \underset{(0.04)}{0.07}\Delta P_{-1}^{IrUs} \qquad \overline{R}^2 = 0.92, \; SE = 0.26, \; t(\hat{\rho}) = 0.15$$

Italy:

$$\Delta P_c^{It} = \underset{(0.003)}{-0.006} + \underset{(0.06)}{0.91}\Delta P_y^{It} + \underset{(0.03)}{0.04}\Delta P_{-1}^{ItGe} + \underset{(0.03)}{0.04}\Delta P^{ItFr} + \underset{(0.02)}{0.03}\Delta P^{ItUs} \qquad \overline{R}^2 = 0.98, \; SE = 0.05, \; t(\hat{\rho}) = 0.40$$

Netherlands:

$$\Delta P_c^{Nl} = \underset{(0.004)}{-0.008} + \underset{(0.07)}{0.86}\Delta P_y^{Nl} + \underset{(0.08)}{0.10}\Delta P_{-1}^{NlGe} + \underset{(0.02)}{0.04}\Delta P^{NlUk} + \underset{(0.04)}{0.10}\Delta P_{-1}^{NlFr} \qquad \overline{R}^2 = 0.92, \; SE = 0.08, \; t(\hat{\rho}) = 1.06$$

USA:

$$\Delta P_c^{Us} = \underset{(0.000)}{0.000} + \underset{(0.05)}{0.94}\Delta P_y^{Us} + \underset{(0.01)}{0.02}\Delta P_{-1}^{UsGe} \qquad \overline{R}^2 = 0.92, \; SE = 0.04, \; t(\hat{\rho}) = 0.32$$

Japan:

$$\Delta P_c^{Ja} = \underset{(0.00)}{0.00} + \underset{(0.07)}{0.78}\Delta P_y^{Ja} + \underset{(0.08)}{0.13}\Delta P_{y_{-1}}^{Ja} + \underset{(0.02)}{0.05}\Delta P^{JaUs} + \underset{(0.02)}{0.04}\Delta P_{-1}^{JaGe} \underset{(0.15)}{-0.20}P_{c_{-1}}^{Ja} + \underset{(0.17)}{0.22}P_{y_{-1}}^{Ja} \qquad \overline{R}^2 = 0.95, \; SE = 0.08, \; t(\hat{\rho}) = -0.57$$

a. *The import price between a home and a foreign country, e.g. Belgium and Germany in the first equation, P^{BeGe}, is defined as $E + P_y^{Ge}$, where E is the exchange rate between Belgium and Germany, defined as the amount of Belgian Francs for one Deutschmark.*

Table 4.8: Estimation results of the employment equation[a]

Belgium:

$$\Delta N^{Be} = \underset{(0.47)}{1.73} + \underset{(0.15)}{0.15}\Delta N^{Be}_{-1} + \underset{(0.06)}{0.35}\Delta Y^{Be} + \underset{(0.07)}{0.18}\Delta Y^{Be}_{-1} - \underset{(0.03)}{0.05}\Delta\lambda^{BeGe} - \underset{(0.06)}{0.26}N^{Be}_{-1} + \underset{(0.01)}{0.02}Y^{Be}_{-1}$$

$\overline{R}^2 = 0.67$, $SE = 0.03$, $t(\hat{\rho}) = -1.56$

Germany:

$$\Delta N^{Ge} = \underset{(0.90)}{1.85} + \underset{(0.10)}{0.55}\Delta N^{Ge}_{-1} - \underset{(0.09)}{0.15}\Delta Wr^{Ge} + \underset{(0.07)}{0.49}\Delta Y^{Ge} - \underset{(0.11)}{0.25}N^{Ge}_{-1} - \underset{(0.07)}{0.08}Wr^{Ge}_{-1} + \underset{(0.07)}{0.11}Y^{Ge}_{-1}$$

$\overline{R}^2 = 0.85$, $SE = 0.03$, $t(\hat{\rho}) = -1.98$

France:

$$\Delta N^{Fr} = \underset{(1.24)}{2.56} - \underset{(0.07)}{0.15}\Delta Wr^{Fr} + \underset{(0.06)}{0.12}\Delta Wr^{Fr}_{-1} + \underset{(0.05)}{0.31}\Delta Y^{Fr} + \underset{(0.06)}{0.20}\Delta Y^{Fr}_{-1} - \underset{(0.01)}{0.04}\Delta\lambda^{FrGe}_{-1} - \underset{(0.10)}{0.16}N^{Fr}_{-1} - \underset{(0.03)}{0.03}Wr^{Fr}_{-1} + \underset{(0.03)}{0.09}Y^{Fr}_{-1} - \underset{(0.000)}{0.001}time$$

$\overline{R}^2 = 0.88$, $SE = 0.01$, $t(\hat{\rho}) = -0.59$

Denmark:

$$\Delta N^{Dn} = \underset{(0.84)}{3.08} + \underset{(0.11)}{0.39}\Delta N^{Dn}_{-1} - \underset{(0.09)}{0.20}\Delta Wr^{Dn} + \underset{(0.06)}{0.39}\Delta Y^{Dn} - \underset{(0.01)}{0.02}\Delta\lambda^{DnUs}_{-1} - \underset{(0.15)}{0.58}N^{Dn}_{-1} - \underset{(0.06)}{0.14}Wr^{Dn}_{-1} + \underset{(0.08)}{0.24}Y^{Dn}_{-1}$$

$\overline{R}^2 = 0.76$, $SE = 0.04$, $t(\hat{\rho}) = -2.40$

United Kingdom:

$$\Delta N^{Uk} = \underset{(2.46)}{9.81} + \underset{(0.12)}{0.53}\Delta N^{Uk}_{-1} - \underset{(0.11)}{0.32}\Delta Wr^{Uk} + \underset{(0.09)}{0.46}\Delta Y^{Uk} + \underset{(0.04)}{0.05}\Delta\lambda^{UkFr} - \underset{(0.03)}{0.03}\Delta\lambda^{UkGe}_{-1} - \underset{(0.09)}{0.40}N^{Uk}_{-1} - \underset{(0.07)}{0.17}Wr^{Uk}_{-1} + \underset{(0.11)}{0.39}Y^{Uk}_{-1} - \underset{(0.001)}{0.005}time$$

$\overline{R}^2 = 0.81$, $SE = 0.05$, $t(\hat{\rho}) = -0.27$

Ireland:

$$\Delta N^{Ir} = -\underset{(0.00)}{0.01} + \underset{(0.14)}{0.31}\Delta N^{Ir}_{-1} - \underset{(0.07)}{0.19}\Delta Wr^{Ir}_{-1} + \underset{(0.07)}{0.28}\Delta Y^{Ir} + \underset{(0.09)}{0.11}\Delta Y^{Ir}_{-1} - \underset{(0.02)}{0.07}\Delta\lambda^{IrUs} + \underset{(0.03)}{0.03}\Delta\lambda^{IrUk}_{-1}$$

$\overline{R}^2 = 0.60$, $SE = 0.07$, $t(\hat{\rho}) = -1.77$

Italy:

$$\Delta N^{It} = \underset{(0.54)}{0.57} + \underset{(0.07)}{0.09}\Delta Wr^{It}_{-1} + \underset{(0.07)}{0.15}\Delta Y^{It} + \underset{(0.06)}{0.14}\Delta Y^{It}_{-1} - \underset{(0.06)}{0.12}N^{It}_{-1} + \underset{(0.01)}{0.04}Y^{Ir}_{-1} + \underset{(0.01)}{0.02}\mathrm{DUM65}$$

$\overline{R}^2 = 0.54$, $SE = 0.04$, $t(\hat{\rho}) = 0.23$

Netherlands:

$$\Delta N^{Nl} = \underset{(0.29)}{1.61} + \underset{(0.10)}{0.61}\Delta N^{Nl}_{-1} + \underset{(0.05)}{0.34}\Delta Y^{Nl} + \underset{(0.05)}{0.10}\Delta Y^{Nl}_{-1} - \underset{(0.04)}{0.26}N^{Nl}_{-1} - \underset{(0.03)}{0.03}Wr^{Nl}_{-1} + \underset{(0.02)}{0.08}Y^{Nl}_{-1}$$

$\overline{R}^2 = 0.88$, $SE = 0.02$, $t(\hat{\rho}) = -2.26$

USA:

$$\Delta N^{Us} = -\underset{(0.09)}{0.10} - \underset{(0.16)}{0.11}\Delta Wr^{Us} + \underset{(0.06)}{0.52}\Delta Y^{Us} + \underset{(0.06)}{0.12}\Delta Y^{Us}_{-1} + \underset{(0.01)}{0.02}\Delta\lambda^{UsJp}_{-1} - \underset{(0.05)}{0.18}N^{Us}_{-1} + \underset{(0.04)}{0.14}Y^{Us}_{-1}$$

$\overline{R}^2 = 0.81$, $SE = 0.04$, $t(\hat{\rho}) = -0.40$

Japan:

$$\Delta N^{Ja} = \underset{(0.72)}{0.86} - \underset{(0.06)}{0.19}Wr^{Jp} + \underset{(0.06)}{0.20}\Delta Y^{Ja} - \underset{(0.08)}{0.08}N^{Ja}_{-1} - \underset{(0.02)}{0.05}Wr^{Ja}_{-1} + \underset{(0.02)}{0.04}Y^{Ja}_{-1}$$

$\overline{R}^2 = 0.38$, $SE = 0.03$, $t(\hat{\rho}) = -0.01$

a. *Real wages, $(W - P_y)$, are presented as Wr in this table. λ, e.g. λ^{BeGe} in the first equation, is defined in the same way as in Table 5.*

we did not impose any restriction on the coefficients of the level variables. In general, the estimated coefficient of the level of (lagged) employment is much higher than the estimated coefficient of the level variable of real GDP indicating that in the long run the effect of real GDP on employment is relatively small. In the equation of the United Kingdom, and to a lesser extent France, we found a significant negative time effect, indicating that technical progress suppresses activity on the labour market. A lagged effect of changes in total employment can be observed in all the equations, except the equations for France, Italy, USA and Japan. This process is remarkably strong in the Netherlands where the coefficient of ΔN_{t-1} is 0.61. The impact of the change in real wages on employment is negative as expected; however, a positive lagged effect was found in France and Italy. The output elasticity on employment is significant (and positive) in each country. Remark that the overall effect of the change in output is rather strong and ranges from 0.29 till 0.53 in the EU-economies. The impact of foreign prices is ambiguous. We found a strong foreign impact in France and Ireland. To improve the fit of the employment equation in Italy we included a dummy, DUM65, which is explained in Appendix B.1. Looking at the $\bar{R}^2$ we see that the fit of the estimated equations in Italy, Ireland and Japan is not so good. Furthermore, notice the high statistic for autocorrelation in three of the ten equations.

4.3.5 The nominal wage equation per private sector employee

It is well-known in macroeconometric modelling that the wage equation is one of the key equations in the model. Usually this equation is adjusted when simulation results are not satisfactory. First of all, we excluded the (lagged) terms of trade , $P_c - P_y$ since the inclusion of these terms yielded a bad simulation performance (see, e.g., the Quest model [10, page 198], for the same findings: 'Another problem may arise in the case of any lasting discrepancy between production and consumption prices. The terms-of-trade coefficient in the wage equation has therefore been set to zero in all countries.'). We started with the following general specification of the equation for nominal wages per employee in the private sector:

$$\begin{aligned}\Delta W_t = \quad & a_0 + a_1 W_{t-1} + a_2 P_{c_{t-1}} + a_3(Y-N)_{t-1} + a_4 U_{t-1} \\ & a_5 \Delta W_{t-1} + a_6 \Delta P_{c_t} + a_7 \Delta P_{c_{t-1}} + a_8 \Delta(Y_t - N_t) + \\ & a_9 \Delta(Y_{t-1} - N_{t-1}) + a_{10}\Delta U_t + a_{11}\Delta U_{t-1}\end{aligned}$$

The selection procedure was based on our priors that $a_1, a_4, a_{10} \leq 0$ and $a_2, a_3, a_6, a_8 \geq 0$. The estimation results can be found in table 4.9. We must remark that the specifications listed in this table are merely equations which were a result of doing static and dynamic simulation exercises and that the general to specific approach was used as a first indication.

Table 4.9: Estimation results of the nominal wage equation[a]

Belgium:

$$\Delta W^{Be} = 0.26 + 0.31\Delta W^{Be}_{-1} + 0.67\Delta P^{Be}_{c} + 0.50\Delta(Y-N)^{Be}$$
(0.57) (0.13) (0.15) (0.19)

$$-0.13W^{Be}_{-1} + 0.21(Y-N)^{Be}_{-1} + 0.11P^{Be}_{c_{-1}} - 0.25U^{Be}_{-1}$$
(0.07) (0.08) (0.10) (0.19)

$\overline{R}^2 = 0.87$, $SE = 0.17$, $t(\hat{\rho}) = -1.55$

Germany:

$$\Delta W^{Ge} = 0.01 + 0.27\Delta W^{Ge}_{-1} + 0.56\Delta P^{Ge}_{c} + 0.68\Delta(Y-N)^{Ge} - 0.44\Delta U^{Ge}_{-1}$$
(0.07) (0.13) (0.18) (0.20) (0.34)

$$-0.06W^{Ge}_{-1} + 0.13(Y-N)^{Ge}_{-1} + 0.06\ \text{DUM70}$$
(0.04) (0.10) (0.01)

$\overline{R}^2 = 0.85$, $SE = 0.13$, $t(\hat{\rho}) = 0.07$

France:

$$\Delta W^{Fr} = 3.00 + 0.36\Delta W^{Fr}_{-1} + 0.86\Delta P^{Fr}_{c} + 0.28\Delta(Y-N)^{Fr} - 0.40\Delta U^{Fr}_{-1}$$
(1.25) (0.10) (0.11) (0.28) (0.44)

$$-0.23W^{Fr}_{-1} + 0.21P^{Fr}_{c_{-1}} + 0.23(Y-N)^{Fr}_{-1}$$
(0.09) (0.10) (0.09)

$\overline{R}^2 = 0.95$, $SE = 0.08$, $t(\hat{\rho}) = -1.21$

Denmark:

$$\Delta W^{Dn} = 1.77 + 0.40\Delta W^{Dn}_{-1} + 0.81\Delta P^{Dn}_{c} + 0.44\Delta(Y-N)^{Dn}$$
(0.70) (0.15) (0.16) (0.17) (0.37)

$$-0.34W^{Dn}_{-1} + 0.33P^{Dn}_{c_{-1}} + 0.43(Y-N)^{Dn}_{-1} - 0.37U^{Dn}_{-1}$$
(0.13) (0.12) (0.20) (0.26)

$\overline{R}^2 = 0.85$, $SE = 0.17$, $t(\hat{\rho}) = -0.14$

United Kingdom:

$$\Delta W^{Uk} = 4.23 + 1.03\Delta P^{Uk}_{c} + 0.57\Delta(Y-N)^{Uk} - 0.88\Delta(Y-N)^{Uk}_{-1}$$
(0.82) (0.06) (0.17) (0.15)

$$-0.75W^{Uk}_{-1} + 0.73P^{Uk}_{c_{-1}} + 0.96(Y-N)^{Uk}_{-1} - 0.32U^{Uk}_{-1}$$
(0.12) (0.13) (0.13) (0.17)

$\overline{R}^2 = 0.95$, $SE = 0.10$, $t(\hat{\rho}) = -0.11$

Ireland:

$$\Delta W^{Ir} = 0.03 + 0.05\Delta W^{Ir}_{-1} + 0.85\Delta P^{Ir}_{c} + 0.39\Delta(Y-N)^{Ir} - 0.20\Delta U^{Ir}_{-1}$$
(0.02) (0.19) (0.17) (0.25) (0.42)

$\overline{R}^2 = 0.78$, $SE = 0.63$, $t(\hat{\rho}) = 0.91$

Italy:

$$\Delta W^{It} = -0.45 + 1.04\Delta P^{It}_{c} - 0.40W^{It}_{-1} + 0.37P^{It}_{c_{-1}} + 0.46(Y-N)^{It}_{-1}$$
(0.17) (0.09) (0.11) (0.11) (0.12)

$\overline{R}^2 = 0.89$, $SE = 0.27$, $t(\hat{\rho}) = -0.70$

Netherlands:

$$\Delta W^{Nl} = 0.81 + 0.24\Delta W^{Nl}_{-1} + 0.82\Delta P^{Nl}_{c} + 0.44\Delta(Y-N)^{Nl}$$
(0.52) (0.12) (0.14) (0.19)

$$-0.26W^{Nl}_{-1} + 0.16P^{Nl}_{c_{-1}} + 0.45(Y-N)^{Nl}_{-1} - 0.20U^{Nl}_{-1}$$
(0.08) (0.09) (0.12) (0.16)

$\overline{R}^2 = 0.93$, $SE = 0.15$, $t(\hat{\rho}) = -1.14$

USA:

$$\Delta W^{Us} = 0.16 + 0.48\Delta P^{Us}_{c} + 0.48\Delta P^{Us}_{c_{-1}} + 0.18\Delta(Y-N)^{Us} - 0.36\Delta U^{Us}$$
(0.11) (0.11) (0.13) (0.15) (0.20)

$$-0.02W^{Us}_{-1} + 0.05(Y-N)^{Us}_{-1} - 0.06U^{Us}_{-1}$$
(0.01) (0.05) (0.15)

$\overline{R}^2 = 0.88$, $SE = 0.04$, $t(\hat{\rho}) = -1.39$

Japan:

$$\Delta W^{Ja} = 2.30 + 0.26\Delta W^{Ja}_{-1} + 0.89\Delta P^{Ja}_{c} + 0.58\Delta(Y-N)^{Ja}$$
(0.70) (0.08) (0.15) (0.17)

$$-0.18W^{Ja}_{-1} + 0.12P^{Ja}_{c_{-1}} + 0.25(Y-N)^{Ja}_{-1}$$
(0.05) (0.07) (0.03)

$\overline{R}^2 = 0.95$, $SE = 0.16$, $t(\hat{\rho}) = -0.35$

Table 4.10: Estimation results of the long term interest rates[a]

Belgium:

$$\begin{array}{llllllll} \Delta RL^{Be} = & 0.014 & -\,0.47RL^{Be}_{-1} & +\,0.34RS^{Be}_{-1} & +\,0.31\Delta RS^{Be} & +\,0.29\Delta RL^{Fr} & +\,0.025\Delta P_c^{Be} & \overline{R}^2 = 0.84 \\ & (0.005) & (0.15) & (0.11) & (0.06) & (0.11) & (0.032) & SE = 0.17 \\ & & & & & & & t(\hat{\rho}) = -0.00 \end{array}$$

Germany:

$$\begin{array}{llllllll} \Delta RL^{Ge} = & 0.028 & -\,0.71RL^{Ge}_{-1} & +\,0.37RS^{Ge}_{-1} & -\,0.21\Delta RL^{Ge}_{-1} & +\,0.24\Delta RS^{Ge} & +\,0.14\Delta RL^{Us} & \overline{R}^2 = 0.90 \\ & (0.005) & (0.10) & (0.06) & (0.08) & (0.03) & (0.07) & SE = 0.08 \\ & & & +0.26\Delta RL^{Ja} & +\,0.083\Delta P_c^{Ge} & & & t(\hat{\rho}) = 0.09 \\ & & & (0.08) & (0.054) & & & \end{array}$$

France:

$$\begin{array}{llllllll} \Delta RL^{Fr} = & 0.005 & -\,0.32RL^{Fr}_{-1} & +\,0.27RS^{Fr}_{-1} & +\,0.33\Delta RS^{Fr} & +\,0.47\Delta RL^{Us} & +\,0.056\Delta P_c^{Fr} & \overline{R}^2 = 0.82 \\ & (0.003) & (0.10) & (0.10) & (0.07) & (0.14) & (0.039) & SE = 0.25 \\ & & & & & & & t(\hat{\rho}) = -1.06 \end{array}$$

Denmark:

$$\begin{array}{lllllll} \Delta RL^{Dn} = & -0.007 & -\,0.60RL^{Dn}_{-1} & +\,0.54RS^{Dn}_{-1} & +\,0.63\Delta RS^{Dn} & +\,0.389\Delta P_c^{Dn} & \overline{R}^2 = 0.85 \\ & (0.005) & (0.13) & (0.15) & (0.07) & (0.070) & SE = 0.61 \\ & & & & & & t(\hat{\rho}) = -1.06 \end{array}$$

United Kingdom:

$$\begin{array}{llllllll} \Delta RL^{Uk} = & 0.010 & -\,0.31RL^{Uk}_{-1} & +\,0.15RS^{Uk}_{-1} & +\,0.28\Delta RS^{Uk} & +\,0.24\Delta RL^{Ge} & +\,0.20\Delta RL^{Fr} & \overline{R}^2 = 0.74 \\ & (0.005) & (0.12) & (0.09) & (0.07) & (0.22) & (0.15) & SE = 0.37 \\ & & & +0.095\Delta P_c^{Uk} & & & & t(\hat{\rho}) = -0.85 \\ & & & (0.040) & & & & \end{array}$$

Ireland:

$$\begin{array}{llllllll} \Delta RL^{Ir} = & 0.003 & -\,0.32RL^{Ir}_{-1} & +\,0.23RS^{Ir}_{-1} & +\,0.23\Delta RS^{Ir} & +\,0.54\Delta RL^{Uk} & +\,0.114\Delta P_c^{Ir} & \overline{R}^2 = 0.71 \\ & (0.005) & (0.12) & (0.11) & (0.10) & (0.16) & (0.039) & SE = 0.56 \\ & & & & & & & t(\hat{\rho}) = -1.78 \end{array}$$

Italy:

$$\begin{array}{llllllll} \Delta RL^{It} = & 0.010 & -\,0.46RL^{It}_{-1} & +\,0.38RS^{It}_{-1} & +\,0.38\Delta RL^{It}_{-1} & +\,0.37\Delta RS^{It} & +\,0.28\Delta RL^{Ge} & \overline{R}^2 = 0.81 \\ & (0.004) & (0.11) & (0.09) & (0.09) & (0.07) & (0.21) & SE = 0.46 \\ & & & +0.22\Delta RL^{Fr} & & & & t(\hat{\rho}) = -0.40 \\ & & & (0.18) & & & & \end{array}$$

Netherlands:

$$\begin{array}{llllllll} \Delta RL^{Nl} = & 0.007 & -\,0.27RL^{Nl}_{-1} & +\,0.20RS^{Nl}_{-1} & +\,0.19\Delta RS^{Nl} & +\,0.23\Delta RL^{Ge} & +\,0.15\Delta RL^{Fr} & \overline{R}^2 = 0.92 \\ & (0.002) & (0.06) & (0.05) & (0.04) & (0.10) & (0.06) & SE = 0.07 \\ & & & +0.15\Delta RL^{Uk} & +\,0.023\Delta P_c^{Nl} & & & t(\hat{\rho}) = 0.32 \\ & & & (0.08) & (0.021) & & & \end{array}$$

USA:

$$\begin{array}{llllllll} \Delta RL^{Us} = & 0.005 & -\,0.51RL^{Us}_{-1} & +\,0.50RS^{Us}_{-1} & -\,0.20\Delta RL^{Us}_{-1} & +\,0.39\Delta RS^{Us} & +\,0.013\Delta P^{Us}_{c_{-1}} & \overline{R}^2 = 0.80 \\ & (0.003) & (0.08) & (0.09) & (0.12) & (0.06) & (0.005) & SE = 0.19 \\ & & & & & & & t(\hat{\rho}) = -2.08 \end{array}$$

Japan:

$$\begin{array}{llllllll} \Delta RL^{Ja} = & 0.012 & -\,0.35RL^{Ja}_{-1} & +\,0.14RS^{Ja}_{-1} & +\,0.19\Delta RS^{Ja} & +\,0.29\Delta RL^{Ge} & +\,0.067\Delta P^{Ja}_{c_{-1}} & \overline{R}^2 = 0.48 \\ & (0.008) & (0.13) & (0.10) & (0.07) & (0.13) & (0.037) & SE = 0.30 \\ & & & & & & & t(\hat{\rho}) = -0.87 \end{array}$$

We included a shock-dummy DUM70 for Germany, which is one in 1970 and zero elsewhere (for an explanation of this dummy we refer to Appendix B.1). Except for Ireland, where the long-run relationship did not prove to be significant, most level variables are included in the equations. In the cases of Germany, France, Italy and Japan we could not find any evidence of a significant negative impact of the unemployment level. The consumer price level did not have any influence in Germany and the USA. The change in labour productivity proved to be an important factor for explaining wages in all countries, except for Italy and to a lesser extent the USA. For seven countries the estimation results of the coefficient of $\Delta(Y - N)$ range from 0.28 till 0.68. Notice, that in the United Kingdom we found a very strong negative impact of a lagged change of labour productivity. Significant effects of lagged growth in wages are observed in all countries except for the United Kingdom, Italy and the USA. The short-run multipliers of consumer price inflation range from 0.48 to 1.04, which is usually considered as satisfactory (see, e.g., Brunia [11] or the Quest model [10]). The absolutu value of the error-correction parameter is high for the United Kingdom indicating a low speed of adjustment to its long term-path. Unemployment persistence effects, reflecting the vulnerability of wages to hysteresis, seems only to be present for a small number of countries. Remark, that for those equations which contain a significant effect of a change in unemployment, the effect is one period lagged. Only for the USA we find a current (negative) effect for the growth of unemployment.

4.3.6 The long term interest rate

Specifications for the long term interest rates in the various countries appeared to be a difficult task. It is clear that during the sample period and the very short term behaviour of the interest rates there were a lot of institutional changes which made it hard to find a good general equation for the whole sample period. Furthermore, the data concerning the short term interest rate were not very reliable during the sixties. For some countries we had to rely on the discount rate as can be seen in appendix B.1. Especially the impact of the short term interest rate in this equation is important since the short term interest rate is a policy variable in our model. Viewing these problems we, finally, adopted a very simple approach. As in the Quest model [5] and Brunia [11] we included RL and RS as level variables in our equation. Furthermore, we added growth variables of the long term interest rate of relevant foreign countries, as specified in table 4.4. In this way we are sure of the direct international monetary linkages. Furthermore, to ensure the linkage between the real part and the monetary part of the model, consumer price inflation and (in first instance) the growth of the budget deficit were included in the estimation process. This last term did not improve the explanatory power of the equation. Hence, we decided to exclude this term from the general equation. The general equation is specified as follows:

$$\Delta RL_t = \quad a_0 + a_1 RL_{t-1} + a_2 RS_{t-1} + a_3 \Delta P_{c_t} + \\ a_4 \Delta RL_{t-1} + a_5 \Delta RS_t + a_6 \Delta RS_{t-1} + a_7(\Delta P_{c_t} - \Delta P_{c_{t-1}}) \\ b_1 \Delta RL_t^{*1} + ... + b_k \Delta RL_t^{*k}$$

The estimation results can be found in table 4.10. In all the equations the sign of a_1 is, as expected, negative and smaller than one in absolute value, indicating a stable relationship for the ECM-mechanism. For the sample period we found some strong effects concerning the direct linkages. The long term interest rate of the USA seems to be important for Germany and France, whereas all the other European Union countries, except Denmark and Ireland, are linked with Germany and/or France. In the equation of Germany we found a strong impact of the long term interest rate of Japan. Remarkable is that in the case of the Netherlands, we found three significant linkage effects: with Germany, France and the United Kingdom. The consumer price inflation was significant in all countries, except for Italy. The change in consumer price inflation did not yield a significant effect in any country. Remark that for the USA and Japan we included lagged consumer price inflation into the equation. This yielded better results than current consumer price inflation.

4.4 The historical tracking performance of the model

In the previous section we focused on reducing errors in single equations. In this section we will investigate the performance of each equation in the complete model. In order to assess the adequacy and validity of the model we present the historical simulation results in this section. To show the performance of the model over the sample period considered, we will perform static and dynamic simulations (see, e.g., Fisher and Wallis [30]). For the static simulation we present for each individual equation the MAE (mean absolute error), which is defined as

$$\text{MAE} := 1/T \sum_{t=1}^{t=T} |p_t - o_t|,$$

the RMSE (root mean square error), which is defined as

$$\text{RMSE} := \sqrt{1/T \sum_{t=1}^{t=T} (p_t - o_t)^2}$$

and the Theil inequality coefficient, which is defined as

$$\text{Theil} := \frac{\sqrt{\sum_{t=1}^{t=T} (p_t - o_t)^2}}{\sqrt{\sum_{t=1}^{t=T} (o_t - o_{t-1})^2}},$$

where p_t is the predicted outcome in the static simulation process and o_t is the observed/actual value for the variable in question at time t, with t ranging from 1963 to 1991. For the dynamic simulation we will just present the Theil inequality coefficients for each individual equation. As argued in Fisher and Wallis [30] static simulation is most appropriate for analysing the historical tracking performance. However, it is our experience that, in practice, dynamic simulation (where residuals accumulate over time) quicker traces certain misspecifications in the model than static simulation. Dynamic simulation also shows some interesting *dynamic* properties of the model, such as robustness.

In table 4.11, the absolute mean of the sample, indicated by $|\bar{X}|$, is compared to the mean absolute error of the static simulation results. Although any classification of the mean absolute errors is arbitrary, Brunia [11] proposes the following standard for the classification of the mean absolute errors. He divides the range of possible values in three sub-ranges. Values less than 0.02 are classified as satisfactory, values larger than 0.04 as poor and values in between as less satisfactory. According to this classification almost all our values are considered satisfactory, except the values for growth in wages and values for GDP inflation for Ireland and Japan and the value for consumer price inflation in Japan. Furthermore, we included the RMSE, the root mean square error, in table 4.11. Considering the fact that the model contains only four exogenous variables per country (labour force, government expenditure, the short term interest rate and the exchange rate) the errors are within range. Comparing these figures with the models for the United Kingdom in Fisher and Wallis [30], we see that these figures have the same order of magnitude [6]. The figures should be used to identify the source of particular difficulties in the model. As the statistics indicate, Ireland, Italy and Japan have higher MAE's and RMSE's than the other countries in the model. This was, however, already implied by our single equation estimates, where some equations of these countries had a low $\bar{R}^2$.

In table 4.12, we present the Theil inequality coefficients. Our static simulation can be compared with a one-step ahead forecast and our dynamic simulation with a one-step ahead forecast for the year 1963, a two-step ahead forecast for the year 1964,..., till a 29-step ahead forecast for the year 1991. If the Theil inequality coefficient is higher than one, the model predicts worse than the so-called naive prediction. This prediction is the prediction of no change. In our static simulation practically all figures are smaller than one. Some values for GDP inflation, growth in wages and the growth in unemployment rate are slightly smaller than one. Only the Theil inequality coefficient of the growth of the unemployment rate in Japan is considerably higher than one. The fact that in Japan the unemployment rate figures are remarkably stable during the sample period explains

[6] One should, however, be very careful in comparing these figures among models. There are many differences among models, such as size, sample, exogenous variables, endogenous variables and so on.

Table 4.11: Static simulation results

Output growth				Growth in Wages		
$\lvert\bar{X}\rvert$	MAE	RMSE		$\lvert\bar{X}\rvert$	MAE	RMSE
0.034	0.010	0.014	Belgium	0.081	0.011	0.014
0.032	0.010	0.012	Germany	0.066	0.011	0.014
0.030	0.008	0.010	France	0.095	0.016	0.019
0.036	0.010	0.012	Denmark	0.091	0.013	0.015
0.028	0.011	0.014	U.K.	0.100	0.015	0.018
0.041	0.012	0.015	Ireland	0.117	0.024	0.030
0.039	0.013	0.016	Italy	0.127	0.018	0.024
0.034	0.011	0.013	Netherlands	0.075	0.017	0.020
0.032	0.011	0.013	USA	0.057	0.007	0.009
0.061	0.012	0.016	Japan	0.080	0.020	0.026

GDP inflation				Growth employment		
$\lvert\bar{X}\rvert$	MAE	RMSE		$\lvert\bar{X}\rvert$	MAE	RMSE
0.048	0.009	0.012	Belgium	0.008	0.006	0.008
0.039	0.006	0.008	Germany	0.011	0.006	0.008
0.065	0.011	0.014	France	0.006	0.003	0.004
0.070	0.008	0.010	Denmark	0.012	0.006	0.008
0.077	0.011	0.013	U.K.	0.012	0.006	0.007
0.082	0.021	0.026	Ireland	0.010	0.006	0.008
0.094	0.016	0.021	Italy	0.008	0.005	0.007
0.048	0.011	0.013	Netherlands	0.011	0.005	0.006
0.049	0.007	0.009	USA	0.020	0.007	0.009
0.047	0.024	0.028	Japan	0.012	0.005	0.005

Consumer price inflation				Long term interest rate		
$\lvert\bar{X}\rvert$	MAE	RMSE		$\lvert\bar{X}\rvert$	MAE	RMSE
0.047	0.010	0.012	Belgium	0.084	0.003	0.004
0.035	0.005	0.006	Germany	0.075	0.002	0.003
0.064	0.013	0.016	France	0.095	0.004	0.005
0.068	0.010	0.013	Denmark	0.119	0.007	0.009
0.074	0.012	0.013	U.K.	0.098	0.004	0.005
0.081	0.015	0.019	Ireland	0.106	0.006	0.008
0.089	0.017	0.022	Italy	0.103	0.005	0.006
0.046	0.012	0.015	Netherlands	0.074	0.002	0.003
0.048	0.008	0.011	USA	0.083	0.003	0.004
0.051	0.021	0.025	Japan	0.073	0.004	0.005

Table 4.12: Theil inequality coefficients

	ΔY		ΔP_y		ΔP_c		ΔW	
	static	dynamic	static	dynamic	static	dynamic	static	dynamic
Belgium	0.53	0.79	0.69	1.31	0.60	1.31	0.72	1.67
Germany	0.50	0.77	0.62	0.89	0.55	0.98	0.61	0.78
France	0.37	0.48	1.03	2.52	0.87	1.84	1.05	2.39
Denmark	0.76	1.13	0.58	1.46	0.76	1.57	0.77	1.93
U.K.	0.57	0.82	0.36	0.77	0.44	0.92	0.45	0.87
Ireland	0.54	0.65	0.64	0.66	0.63	0.87	0.83	0.77
Italy	0.58	0.67	0.79	1.46	0.83	1.43	0.67	1.20
Netherlands	0.60	0.81	0.77	1.31	0.72	1.20	0.84	1.26
USA	0.52	0.74	0.74	1.41	0.80	1.44	0.78	1.60
Japan	0.58	0.60	1.01	1.34	0.86	1.30	1.00	1.53

	RL		ΔRL		ΔN		ΔU	
	static	dynamic	static	dynamic	static	dynamic	static	dynamic
Belgium	0.40	0.45	0.34	0.44	0.77	1.12	1.01	1.47
Germany	0.31	0.37	0.24	0.41	0.59	1.01	1.03	1.76
France	0.41	0.70	0.34	0.47	0.49	0.70	0.65	0.92
Denmark	0.43	0.81	0.29	0.41	0.47	1.01	0.63	1.13
U.K.	0.42	0.71	0.33	0.33	0.48	0.86	0.66	1.40
Ireland	0.56	0.76	0.41	0.41	0.51	0.65	0.54	0.70
Italy	0.39	0.56	0.37	0.37	0.52	0.70	0.87	1.16
Netherlands	0.30	0.47	0.24	0.24	0.65	1.26	0.68	1.31
USA	0.40	0.52	0.35	0.35	0.55	0.76	0.67	0.93
Japan	0.68	0.95	0.51	0.51	0.62	0.61	2.53	2.47

most of the difficulties. In our dynamic simulations, most figures of inflation and wages are above one, but most figures of output growth, long term interest rate and growth of employment are all less than one. This is, of course, not remarkable because in dynamic simulation errors accumulate. On average, the Theil inequality coefficients of the dynamic simulation results are less than twice the Theil inequality coefficients for the static simulation exercise. If we look at the Theil inequality coefficients as published in Fisher and Wallis [30], we see that for most U.K. models in the static simulation many coefficients are above one. Therefore, the overall impression from the shown statistics is that the model is capable of reproducing the most important developments during the sample period.

4.5 Shock analysis

To show the dynamic properties of the model we applied several shocks to the model. The shocks are similar to the shocks Whitley [83] applies to several large multi-country models. The first five shocks applied to our model will be compared with the outcomes Whitley obtains in his analysis. The models considered in Whitley [83] are the EU's model (QUEST) as operated by the Deutsches Institut fur Wirtschaftsforschung (DIW); the model of the National Institute of Economic and Social Research and jointly operated with the London Business School (GEM); the model used by OFCE/CEPII (MIMOSA); the Oxford Economic Forecasting model (OEF); and the OECD's Interlink model (OECD). These models are much larger in size than our small multi-country model. Beforehand, we can already stress that the striking difference with our model and the large multi-country models is the representation of the aggregate demand equation, which in our model equals aggregate supply. The fact that GDP in our model is expressed by only one estimated equation, instead of separately modelled sub-categories like consumption, investment, exports and imports, explains most of the differences. However, in order to check the qualitative properties of the model, we compare the obtained results with the outcomes of these models. Whitley's analysis [83] compares the four major European economies, Germany, France, Italy and the United Kingdom. We will present his findings together, with the findings of our model.

4.5.1 Single country shocks

First, we give an impression of some country-specific developments. We analyse the effects of a permanent shock originating in a domestic country on the domestic variables of that country. In the case of a linear model, the outcomes of applying a certain shock are base independent. Therefore, it does not matter in which year the shock is applied. We used for each shock the year 1963 as the starting point. Now, for each country separately, we consider the following four shocks:

(1) Fiscal shock: a 1% of GDP shock to government expenditure.
Expenditure is raised by 1% of GDP of its base value in the years 1963-1991. The simulation is carried out with fixed real interest rates. The real interest rate in our model is fixed as follows: a new variable is introduced which replaces the term $(RL_{t-1} - \Delta P_{yt})$ in the GDP equation. This new variable keeps his historical value throughout the simulation exercise.
(2) Wage shock: a 1% wage shock.
The wage variable is made exogenous, which is performed by skipping the wage equation in each country model. This exogenous wage variable is raised then by one percent of its

base value throughout the period 1963-1991. Real interest rates are kept fixed and wage costs are held constant in all other countries.
(3) Monetary shock: a 1% nominal short-term interest rate shock.
The nominal short-term interest rate is raised by 1% point, throughout the period 1963-1991.
(4) Exchange rate shock: the dollar exchange rate is reduced by 10% below base, in every year of the period 1963-1991, for each country separately. Nominal interest rates are kept fixed. In our model several exchange rates between countries are modelled with the USA as linking country. For instance, the exchange rate between Germany and Belgium is modelled as $E^{GeBe} = E^{GeUs}E^{UsBe}$ (E^{GeBe}, the exchange rate between Germany and Belgium, is defined as the amount of German Deutschmark for one Belgian Franc). In our experiment a 10% fall of the effective exchange rate was simulated by raising E^{GeUs} by 11.11%; hence, by depreciating the US Dollar vis-à-vis the Deutschmark .

The simulation experiments in Whitley [83] are conducted on a forecast base of each model over a 6-year time horizon. The figures presented in his study are of year 1, year 3, year 5 and year 6. For each simulation experiment, Whitley [83] presents the corresponding change in output and GDP-price. Remark that, for some experiments, the figures in year 6 can be considered as long-run values. This is probably the case if the differences between the figures in years 5 and 6 are small. For comparative reasons we will present the figures for our model for the same years.

(1) Single-country fiscal shocks.
The results are shown in table 4.13. First, the four countries: Germany, France, Italy and the United Kingdom are presented, as listed in Whitley's paper [83]. The outcomes of our model are presented under the heading SLIM. At the end of table 4.13 we have listed the simulated figures for the other countries of our model. In our model, expanding G raises aggregate demand/output. This rise in output will raise prices, wages and employment. In most countries, the long term interest rate depends on the consumer price inflation which implies that the long term interest rate also increases for those countries (tightening monetary policy).
There is one important major difference between the simulation exercise of the existing multi-country models and our model. In our model we estimated the effect of the impact of government expenditure on output, whereas in the multi-country models this effect is simulated by raising the government expenditure component as part of the GDP identity equation. As a result, our model shows much more differentiation between the output responses of the various countries than the output responses of the large scale models in Whitley [83].
In all simulations output increases and almost all figures show in year 6 an effect which

is higher than 0.3. Exceptions are the OECD model of Germany, the GEM model in the United Kingdom and Ireland in the SLIM model. In our model the effect of a 1% increase of government expenditure is highest in the three major economies, Germany, USA and Japan. Weak responses are found for France, the United Kingdom and Italy. A negative effect in year 5 and 6 is found for Ireland. This effect is easy to explain if we go back to the estimation result of this equation. It is the only country where we could not find any evidence of a positive effect of the level variable G. This aspect clarifies that there is, even, an undershooting effect of the baseline in year 5 and year 6. Simulation shows that in the long-run this effect will peter out and become zero for GDP in Ireland. A possible 'explanation' for this result might be that the Irish economy has done relatively well in the second half of the eighties in spite of considerable fiscal consolidation. The zero effect in year 1 of GDP in Denmark and Japan is explained by the fact that we could not find a significant effect of ΔG in our estimated equation. In all countries, except Ireland, output will be raised permanently, because the level variable G occurs in the GDP-equation.
The development of prices looks adequately and, more or less, coincides with the findings of the large models. If we compare for each country the ratio of GDP-price response and GDP-response we find in our model relatively high figures for Italy and France.

(2) Single-country wage shocks.
In table 4.14 the results of this wage shock are presented. The only way by which wages influence output in the GDP-equation is the term of the real exchange rate. The outcome of this simulation exercise gives an indication of the effect of this variable on output for each country. In the large multi-country models there are many more ways by which wages affect output (i.e., wealth effects, consumption effects). The large models find, on average, somewhat stronger quantitative effects than our model. Qualitatively, the effect of a rise in wages on output is the same. We could not find any effects for Denmark, USA and Japan. This is due to the fact that in our estimations there was no evidence of any significant effect for the real exchange rate variable.
The effect of wages on prices is much stronger than the effect of wages on output. The results of our model coincide with the findings of the large models. The small change in the price level between years 5 and 6 indicates that the price-level effects have settled down sufficiently after year 6. This observation makes it possible to interpret the values of the price level in year 6 as long-run responses. Our model finds strong long-run responses for USA and Italy and low responses for Belgium and Japan.

(3) Single-country monetary shocks.
The results of this experiment are shown in table 4.15. If we look at our estimation results, we see that a rise in the nominal short term interest rate directly affects output in the United Kingdom, Ireland and the USA. In the other countries the influence of short term

Table 4.13: Fiscal shock-single country simulations. An increase of government expenditure by 1% of GDP, with fixed real interest rates (percentage difference from base)

	GDP				GDP Prices			
	year 1	year 3	year 5	year 6	year 1	year 3	year 5	year 6
	Germany							
DIW/QUEST	1.61	0.83	0.63	0.75	0.03	1.17	1.83	1.98
GEM	0.87	0.62	0.57	0.56	0.06	0.35	0.52	0.58
OECD	0.60	0.18	0.10	0.05	0.55	1.37	2.00	2.17
OEF	0.77	0.80	0.77	0.72	0.34	0.92	1.06	1.07
MIMOSA	1.07	0.90	0.66	0.61	0.14	0.90	1.60	1.92
SLIM	1.82	1.44	1.36	1.43	-0.01	0.83	1.21	1.35
	France							
DIW/QUEST	1.50	1.74	1.48	1.40	-0.55	-0.58	-0.12	0.05
GEM	0.58	0.63	0.62	0.61	0.14	0.32	0.54	0.63
OECD	0.50	0.87	0.82	0.71	0.19	0.68	1.29	1.59
OEF	0.98	1.02	0.96	0.89	-0.24	0.25	0.86	1.20
MIMOSA	1.04	1.13	1.10	1.09	-0.13	-0.20	-0.02	0.14
SLIM	0.40	0.34	0.33	0.33	0.08	0.33	0.65	0.77
	Italy							
DIW/QUEST	1.20	1.11	0.86	0.85	-0.10	1.48	2.74	3.12
GEM	0.48	0.57	0.56	0.53	0.06	0.30	0.50	0.60
OECD	0.73	0.53	0.47	0.48	0.30	0.89	1.10	1.12
OEF	1.17	0.97	0.75	0.67	0.47	1.70	2.26	2.42
MIMOSA	1.07	1.41	1.34	1.28	0.03	0.39	1.49	2.22
SLIM	0.21	0.47	0.65	0.71	0.10	0.73	1.67	2.18
	United Kingdom							
DIW/QUEST	1.41	1.17	1.03	1.17	-0.08	1.03	1.58	1.61
GEM	0.50	0.34	0.27	0.29	0.09	0.68	0.89	0.88
OECD	0.55	0.73	0.47	0.45	0.16	1.04	1.94	2.21
OEF	0.98	1.50	1.24	1.18	0.29	1.72	3.01	3.10
MIMOSA	0.79	0.73	0.57	0.46	0.28	0.84	1.30	1.48
SLIM	0.18	0.29	0.31	0.31	-0.10	0.09	0.30	0.38
	other countries in SLIM							
Belgium	1.10	1.05	1.02	1.00	0.12	1.10	1.65	1.82
Denmark	0.00	0.77	1.02	1.03	0.00	0.02	0.78	1.35
Ireland	0.71	0.05	-0.21	-0.07	-0.22	-0.05	0.14	0.11
Netherlands	1.13	0.96	0.87	0.86	0.00	0.61	1.06	1.24
USA	1.23	1.89	1.35	1.27	0.00	0.51	1.13	1.38
Japan	0.00	0.94	1.59	1.88	0.00	0.35	1.13	1.51

Table 4.14: Wage shock-single country simulations. Shock of 1% to wage compensation, with fixed real interest rates (percentage difference from base)

	GDP				GDP Prices			
	year 1	year 3	year 5	year 6	year 1	year 3	year 5	year 6
	Germany							
DIW/QUEST	-0.13	-0.34	-0.23	-0.28	0.53	1.26	1.20	1.22
GEM	0.15	-0.07	-0.12	-0.12	0.10	0.35	0.47	0.51
OECD	-0.50	-0.05	-0.30	-0.12	0.60	0.78	0.75	0.73
OEF	0.09	0.03	-0.02	-0.03	0.60	0.70	0.69	0.70
MIMOSA	-0.12	-0.18	-0.12	-0.11	0.42	0.68	0.77	0.80
SLIM	0.03	-0.01	-0.01	-0.01	0.45	0.63	0.67	0.68
	France							
DIW/QUEST	-0.28	-0.19	-0.13	-0.13	0.83	0.98	0.99	1.01
GEM	-0.11	-0.22	-0.17	-0.15	0.56	0.88	0.89	0.89
OECD	-0.25	-0.02	-0.02	-0.03	0.69	0.89	0.89	0.88
OEF	-0.03	-0.13	-0.16	-0.17	0.47	0.73	0.79	0.81
MIMOSA	0.02	0.05	0.05	0.03	0.39	0.55	0.61	0.63
SLIM	-0.12	-0.12	-0.09	-0.08	0.57	0.83	0.85	0.86
	Italy							
DIW/QUEST	-0.15	-0.18	-0.14	-0.15	0.86	1.00	0.97	0.98
GEM	0.00	-0.16	-0.29	-0.33	0.07	0.43	0.63	0.69
OECD	-0.11	-0.15	-0.10	-0.10	0.70	0.62	0.55	0.52
OEF	0.03	-0.20	-0.24	-0.25	0.50	0.72	0.75	0.77
MIMOSA	-0.04	-0.17	-0.14	-0.12	0.53	0.91	0.99	0.99
SLIM	-0.05	-0.07	-0.06	-0.06	0.54	0.86	0.93	0.94
	United Kingdom							
DIW/QUEST	-0.46	-0.53	-0.08	0.05	0.86	1.25	1.15	1.09
GEM	-0.01	-0.25	-0.20	-0.08	0.09	0.69	0.86	0.89
OECD	-0.12	-0.17	-0.08	-0.09	0.65	0.83	0.83	0.82
OEF	0.11	0.11	-0.03	-0.06	0.36	0.94	0.88	0.86
MIMOSA	-0.04	-0.19	-0.26	-0.29	0.66	0.90	0.94	0.93
SLIM	0.00	-0.01	-0.02	-0.02	0.00	0.22	0.53	0.66
	other countries in SLIM							
Belgium	-0.03	-0.03	-0.02	-0.02	0.28	0.44	0.48	0.50
Denmark	0.00	0.00	0.00	0.00	0.38	0.65	0.73	0.75
Ireland	-0.07	-0.03	0.02	0.01	0.71	0.91	0.87	0.86
Netherlands	-0.18	-0.11	-0.08	-0.07	0.60	0.64	0.65	0.66
USA	0.00	0.00	0.00	0.00	0.00	0.53	1.08	1.20
Japan	0.00	0.00	0.00	0.00	0.74	0.51	0.47	0.47

nominal interest rates on output is indirect. A rise in the nominal short interest rates raises the long term interest rate and this affects output. In our model, as in most large multi-country models, a rise in the nominal short term interest rates lowers output. If we look at the responses on output we find, with the exception of France in year 6, figures which are between zero and minus one. The differences between the countries are modest and certainly not as large as in the government expenditure experiment. We find no effect at all for Japan, because interest rates were not included in the GDP-equation for Japan. In general the effect on prices is ambiguous in the first three years and negative in year five and six. An outlier is (again) Ireland which has a positive price development. The reason for this can be found in the GDP inflation equation for Ireland where we found a very strong negative effect of the change in output minus output trend. The United Kingdom starts initially with an overshooting effect, but in the long-run the effects on prices are negative. The positive effect in the short run can be traced back to the wage equation where we found a strong negative effect of a change in lagged labour productivity. Note, that in most cases the values in year 6 cannot be interpreted as long-run values, because most effects have not settled down after year 6. Remark, that also for the large multi-country models there is no clear evidence that a rise in the nominal interest rate should lower prices. For all the four major EU-economies, there is at least one model which predicts a rise in prices.

(4) Single-country exchange rate shocks.
The results of this simulation are shown in table 4.16. Through various channels, such as the GDP equation, the price equations (domestic consumer prices and GDP prices) and the employment equation, the exchange rate influences all the variables in our model. This experiment therefore gives an idea of the impact of these variables in our model. The output response for Germany and Denmark is low. We find strong effects for France and the Netherlands. Qualitatively, the output responses are more or less comparable to the large models (except for Germany).
The price responses between the countries in the model are quite different. For the countries France, Italy, Belgium, Ireland and the Netherlands we find strong price responses. Full homogeneity of prices in the medium term is present in the countries France, Italy and Ireland. For the large countries Germany, the United Kingdom and Japan we found very small price responses contrary to the large multi-country models in the literature. There are several reasons for these small responses. First of all, we must stress that the exchange rate in our model appears only in differences, so that in the long-run there is a tendendency that effects will return back to the baseline. Secondly, in some aggregate demand equations we found only limited real exchange rate effects. Remark, however, that also for the multi-country models, full homogeneity is present in only a small number of the country models.

Table 4.15: Monetary shock single country simulations. Shock of 1% rise in the nominal short term interest rate for 6 years (percentage difference from base)

	GDP				GDP Prices			
	year 1	year 3	year 5	year 6	year 1	year 3	year 5	year 6
	Germany							
DIW/QUEST	-0.18	-0.92	-0.55	0.36	0.00	-0.19	-0.76	-0.93
GEM	-0.24	-0.99	-1.37	-1.50	-0.01	-0.25	-0.65	-1.22
OECD	-0.50	-0.42	-0.64	-0.53	-0.32	-1.59	-1.87	-1.83
OEF	-0.28	-0.50	-0.64	-0.71	-0.08	-0.32	-0.05	0.16
MIMOSA	-0.38	-0.71	-0.68	-0.65	0.07	0.18	0.36	0.51
SLIM	0.00	-0.44	-0.50	-0.48	0.00	-0.07	-0.27	-0.35
	France							
DIW/QUEST	-0.19	-0.50	-0.52	-0.53	0.06	0.20	0.16	0.14
GEM	-0.09	-0.36	-0.52	-0.58	-0.02	-0.14	-0.30	-0.40
OECD	-0.31	-0.47	-0.58	-0.61	-0.04	-0.13	-0.12	-0.07
OEF	-0.03	-0.07	-0.06	-0.06	0.01	-0.01	-0.05	-0.07
MIMOSA	-0.21	-0.21	-0.20	-0.20	0.12	0.24	0.28	0.27
SLIM	0.00	-0.36	-0.81	-1.03	0.00	-0.11	-0.52	-0.88
	Italy							
DIW/QUEST	-0.08	-0.29	-0.10	-0.12	0.00	-0.15	-0.44	-0.49
GEM	-0.07	-0.54	-0.88	-0.99	0.00	-0.15	-0.46	-0.65
OECD	-0.07	-0.48	-0.62	-0.73	0.06	0.08	0.17	0.23
OEF	-0.12	-0.52	-0.66	-0.74	-0.02	-0.50	-0.67	-0.53
MIMOSA	0.06	0.14	0.17	0.18	0.07	0.56	1.30	1.70
SLIM	0.00	-0.20	-0.36	-0.23	0.00	-0.35	-1.04	-1.24
	United Kingdom							
DIW/QUEST	-0.07	-0.41	-0.29	-0.25	0.00	-0.07	-0.37	-0.42
GEM	-0.20	-0.77	-0.89	-0.87	-0.02	-0.60	-1.60	-2.00
OECD	-0.20	-0.61	-0.56	-0.57	-0.05	-0.52	-0.84	-0.70
OEF	-0.26	-1.17	-1.42	-1.36	-0.01	-0.34	-1.28	-1.69
MIMOSA	-0.02	-0.05	-0.05	-0.07	0.07	0.33	0.64	0.78
SLIM	0.00	-0.43	-0.53	-0.55	0.00	0.06	-0.31	-0.49
	other countries in SLIM							
Belgium	0.00	-0.25	-0.60	-0.73	0.00	-0.07	-0.41	-0.65
Denmark	0.00	-0.15	-0.24	-0.26	0.00	0.00	-0.15	-0.27
Ireland	0.00	-0.53	-0.65	-0.60	0.00	0.18	0.25	0.20
Netherlands	0.00	-0.10	-0.33	-0.45	0.00	0.00	-0.09	-0.18
USA	0.00	-0.99	-0.62	-0.46	0.00	-0.12	-0.44	-0.55
Japan	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 4.16: Exchange rate shock single country simulations. A 10% fall in the nominal exchange rate, with fixed real interest rates (percentage difference from base)

	GDP				GDP Prices			
	year 1	year 3	year 5	year 6	year 1	year 3	year 5	year 6
	Germany							
DIW/QUEST	2.98	3.55	1.42	0.26	-0.23	5.12	9.93	11.23
GEM	1.56	1.45	1.15	1.01	-0.04	2.06	3.68	4.31
OECD	0.98	1.16	0.25	0.12	1.71	8.59	12.95	13.48
OEF	1.30	1.24	1.01	0.89	0.01	2.71	3.14	3.27
MIMOSA	1.62	2.53	1.99	1.62	-0.38	1.82	3.87	4.69
SLIM	-0.37	0.10	-0.09	-0.12	0.27	0.64	0.53	0.38
	France							
DIW/QUEST	1.04	0.37	0.70	0.83	1.19	3.75	2.66	2.11
GEM	0.47	0.35	0.24	0.20	0.49	3.71	5.50	6.06
OECD	-0.01	0.96	1.39	1.34	1.06	5.51	7.18	7.55
OEF	0.17	0.60	0.10	-0.07	0.79	4.39	6.72	7.53
MIMOSA	0.69	0.59	0.46	0.43	0.98	3.16	4.62	5.24
SLIM	2.52	2.35	2.16	2.08	1.23	4.81	8.30	9.51
	Italy							
DIW/QUEST	1.20	0.57	-0.30	-0.66	-0.10	6.54	9.50	9.94
GEM	1.78	1.74	1.69	1.59	1.01	3.19	4.89	5.68
OECD	0.39	0.10	0.08	0.37	2.74	9.52	8.45	7.45
OEF	0.53	1.32	0.02	-0.52	0.64	5.73	9.60	10.60
MIMOSA	1.52	1.12	0.75	0.57	-0.29	1.95	3.79	4.68
SLIM	2.11	0.86	-0.36	-0.58	3.11	9.11	10.72	10.22
	United Kingdom							
DIW/QUEST	1.34	0.06	-0.66	-0.68	1.41	6.67	9.60	10.27
GEM	0.80	0.63	-0.07	-0.11	0.66	8.42	11.58	11.63
OECD	0.30	1.60	1.00	0.62	1.36	5.08	8.11	8.78
OEF	0.40	0.81	0.82	0.51	-0.07	4.97	8.07	8.90
MIMOSA	0.28	1.37	1.25	1.04	0.57	3.00	4.76	5.54
SLIM	0.49	0.37	0.23	0.17	-0.28	0.46	0.92	1.04
	other countries in SLIM							
Belgium	1.62	1.28	1.06	0.82	2.90	6.28	7.50	7.68
Denmark	0.03	0.19	0.15	0.10	0.26	2.17	3.08	3.20
Ireland	1.51	2.24	0.20	-0.27	3.11	11.32	13.21	13.27
Netherlands	2.57	2.33	1.71	1.32	1.47	7.56	7.55	7.22
Japan	0.00	0.00	0.00	0.00	0.80	1.42	0.05	-0.31

We have now examined the two key variables in the model, output and prices for each country separately. To see whether the effects of the other key endogenous variables, such as wages, employment and the unemployment rate are similar as in the large multi-country models, we adopted an approach as developed by Hickman [43]. This approach is also adopted by Whitley [83, 84]. They suggest to decompose price-output responses (the inverse of the aggregate supply elasticity) into various ratios of key endogenous variables:

$$\Delta P/\Delta Y = \Delta P/\Delta W \cdot \Delta W/\Delta U \cdot \Delta U/\Delta N \cdot \Delta N/\Delta Y$$

The ratios of the key endogenous variables can be specified as follows (where Δ denotes percentage deviation from the base simulation):
$\Delta P/\Delta Y$: the inverse of the aggregate supply elasticity.
$\Delta P/\Delta W$: the ratio of prices to wages.
$\Delta W/\Delta U$: demand effect on wages.
$\Delta U/\Delta N$: labour force participation.
$\Delta N/\Delta Y$: movements in productivity.
One would expect that a positive sign of $\Delta P/\Delta Y$ is determined by a positive sign of $\Delta P/\Delta W$ and $\Delta N/\Delta Y$ and a negative sign of $\Delta U/\Delta N$ and $\Delta W/\Delta U$. We calculated these figures, just as Whitley [83, 84] did, for our first experiment (a rise in government expenditure). The results are presented for year 6 (as percentage deviation from the base simulation) in table 4.17. These figures show that the contributions of most endogenous variables are qualitatively within range and coincide with the figures as presented in Whitley's paper. As stressed in Whitley [83, 84] the figures should be treated with care, but, as he claims, it can be useful *in some cases* to highlight particular differences in structure of certain models. Most figures seem quite acceptable with some outliers. In Ireland $\Delta P/\Delta Y$ is negative and this can be traced back to the fact that in Ireland there is no level variable of G, government expenditure, in the equation. As can be seen from table 4.5, ΔY was negative which indicates an undershooting effect. This effect influenced all other ratios of Ireland in the table 4.17. For the other nine countries, table 4.17 shows some interesting properties. If we look at the averages of year 6 and compare them with the averages of Whitley [83], we see that our country models exhibit weaker inflationary effects from a demand expansion, implying a flatter aggregate supply schedule in the medium term. In the long-run the aggregate supply elasticity is lower and equals the elasticity as given by Whitley [83]. Furthermore, a striking difference with the multi-country models is the lower ratio of movements in productivity, implying that an aggregate demand shock has only limited power to raise employment. This is more in line with the history of the past 40 years, where productivity has gone up more or less steadily, and employment has shown little trend (see Blanchard [6]). In our model, a shock of government expenditure affects employment mainly in the short term. A major reason for this finding is that we did not impose any restriction in the employment equation. Strongly related with the ratio of

Table 4.17: Contributions to the aggregate supply elasticity: year 6

	$\Delta P/\Delta W$	$\Delta W/\Delta U$	$\Delta U/\Delta N$	$\Delta N/\Delta Y$	$\Delta P/\Delta Y$	AS
Belgium	0.65	-12.1	-1	0.23	1.83	0.55
Denmark	0.52	-18.6	-1	0.14	1.31	0.76
Ireland	2.40	0.9	-1	0.42	-1.49	-0.67
Netherlands	0.60	-12.4	-1	0.19	1.44	0.69
USA	0.74	-2.0	-1	0.74	1.09	0.92
Japan	0.47	-24.6	-1	0.07	0.81	1.24
Germany	0.62	-6.1	-1	0.25	0.95	1.06
France	0.74	-5.8	-1	0.54	2.30	0.43
Italy	0.90	-10.1	-1	0.34	3.05	0.33
United Kingdom	0.71	-2.1	-1	0.82	1.23	0.81
Average of year 6 [a]	0.66	-10.4	-1	0.37	1.56	0.75
Average of year 29 [b]	0.80	-14.1	-1	0.39	2.81	0.47
Average in [83] [c]	0.85	-5.09	-0.74	0.73	2.15	0.47

a. Results are subject to rounding. The contribution of Ireland is excluded from the average. Furthermore, all variables are measured as percentage difference from base (except unemployment rate, absolute difference from base). The value for the aggregate supply elasticity is indicated by AS and is defined as $1/(\Delta P/\Delta Y)$.

b. These are the averages (excluded Ireland) of year 29, which give a good indication of the long-run properties of the model.

c. These are the averages of year 6 as published by Whitley [83, 84].

movement in productivity is the high (negative) elasticity of the demand effect on wages. Remark also that in the medium term (year 6) the ratio of prices to wages is lower than the average published by Whitley [83]. In the long term (year 28) the elasticity is higher which is an indication that most effects have not settled down after 6 years.

4.5.2 International linkages

An important aspect of the model is that it is capable to show various external effects when there is a change of a domestic macroeconomic policy. In this section we will concentrate on these spillover effects. One of our goals was to construct a model that contains strong international linkages. To show this aspect of the model, various shocks will be applied. To save space we restrict ourselves to five shocks. With these shocks we analyse the spillover effects to other EU-countries from a single country's expansion.
The five shocks we apply are the following:
(1) Fiscal shock USA:
2% GDP shock to government expenditure, throughout, in the USA is applied to the model. Real interest rates are kept fixed for all the countries.
(2) Fiscal shock Germany:
1% GDP shock to government expenditure, throughout, in Germany is applied. We analyse the effects of most of the endogenous variables of the model. In order to compare certain effects with the shock in (1) we keep the real interest rates fixed.
(3) Fiscal shock France:
2% GDP shock to government expenditure, throughout, in France is applied. Real interest rates are kept fixed.
(4) Monetary shock in Germany:
2% short-term nominal interest rate shock in Germany. The nominal short-term interest rate of Germany is raised 2% point, throughout the period 1963-1991.
(5) Exchange rate shock to the US dollar vis-à-vis all other currencies: A depreciation of the US dollar, throughout, by 10% vis-à-vis all other currencies. This experiment is done in the same way as explained in the single country experiments but now we depreciate the US dollar against all currencies at once. As in the Whitley experiment [83], we keep nominal interest rates fixed. Since Whitley reports no explicit numbers of the last four shocks, we will present these simulation results in figures 4.19-4.20.

(1) Fiscal shock USA.
In table 4.18 we show the results of a fiscal shock in the USA and compare them with the outcomes of the large scale models. The effect of the USA on other economies in our model corresponds (roughly) during the first three years with the findings in Whitley [83]. Again

we have to stress that in each country foreign influences are modelled as differences (and not in levels), so, in the long-run the effect on foreign output will return to its baseline. This model property corresponds to the output figures for our model in years five and six. Remarkable is that seven countries in our model show cyclical behaviour in the GDP figures, which is due to the cyclical output response of the USA. In our model, the first year effect of an increase in government expenditure in the USA on the foreign countries lies between 0.12 % and 0.46 % of USA GDP-output. The United Kingdom profits most, and Italy least.

The development of the prices is qualitatively also comparable with the outcomes of the other models. On average, quantitatively, the price responses are weaker than predicted by the other multi-country models. Most price responses are lower than 1% in year 6 which is explained by the low output response in the medium term, which surpresses the price development.

(2) Fiscal shock Germany.

Considering the size of the effect (first year GDP output of Germany is higher than first year GDP output in the USA; see our previous experiment) we find various interesting results. First of all the first graph in figure 4.19 shows that small open economies like the Netherlands, Denmark and Belgium, are heavily affected by a German expenditure shock. If we again compare the effect in the first year we find that the effect of a German expenditure shock on the foreign economies lies between 0.07 % and 0.53 % of German GDP-output in the first year. Remark that also in this experiment there is some cyclical behaviour in the medium term and long term. In the long-run, most effects peter out to a very small (positive) value.

As the second graph in figure 4.19 shows, the development of prices is quite strong in most EU-economies. Only the prices in the USA, Japan, U.K., and Ireland seem to be less affected by the German government expenditure shock. Note also that most countries which are affected by the shock have their highest price response around year four and five. Note that the output response in Italy in the first graph in figure 4.19 was rather low whereas the price response in the second graph of figure 4.19 is relatively strong, which is due to the interaction terms ΔP^{ItGe} in the two price equations of Italy.

(3) Fiscal shock France.

The third graph in figure 4.19 shows the results of a fiscal shock originating in France. Again we find pretty large effects, in the short run, in all the other countries. If we again compare the effect in the first year we find effects between 0.03 % and 0.87 % of French GDP-output in the first year. Belgium and the Netherlands profit most from an output shock in France.

Concerning the other endogenous variables, which are not presented here, we can note that

Table 4.18: USA fiscal shock government spending. Permanent shock of 2% of GDP rise in government expenditure, fixed real interest rates (percentage difference from base).

	year 1	year 3	year 5	year 6	year 1	year 3	year 5	year 6
	Germany		GDP			GDP Prices		
DIW/QUEST	0.42	0.62	0.37	0.47	0.01	0.40	0.99	1.18
GEM	0.42	0.26	0.15	0.14	0.00	0.28	0.58	0.73
OECD	0.31	0.14	0.13	0.13	0.34	1.13	2.34	3.06
OEF	0.21	0.63	0.77	0.80	0.01	0.31	0.70	0.92
MIMOSA	1.12	0.74	0.76	1.06	0.06	0.92	1.57	1.98
SLIM	0.87	0.42	-0.20	-0.14	0.00	0.45	0.44	0.36
	France							
DIW/QUEST	0.27	0.61	0.44	0.47	-0.10	-0.13	0.22	0.30
GEM	0.14	0.13	0.09	0.09	0.04	0.35	0.69	0.85
OECD	0.14	0.22	0.27	0.29	0.11	0.54	1.30	1.83
OEF	0.08	0.23	0.28	0.29	0.05	0.40	0.77	0.97
MIMOSA	0.88	0.75	0.91	1.17	-0.12	0.18	0.79	1.28
SLIM	0.42	0.19	-0.20	-0.20	0.07	0.40	0.77	0.82
	Italy							
DIW/QUEST	0.26	0.54	0.40	0.40	-0.01	0.42	1.23	1.51
GEM	0.16	0.18	0.19	0.20	0.07	0.35	0.62	0.77
OECD	0.24	0.10	0.09	0.06	0.16	0.78	1.65	2.22
OEF	0.09	0.24	0.24	0.20	0.02	0.40	0.94	1.23
MIMOSA	0.69	0.74	0.79	1.00	-0.34	0.05	0.95	1.46
SLIM	0.28	0.21	-0.02	-0.02	0.14	0.80	1.09	1.08
	United Kingdom							
DIW/QUEST	0.23	0.32	0.10	0.21	-0.04	0.33	0.85	0.95
GEM	0.17	0.01	-0.08	-0.04	0.05	0.79	1.32	1.47
OECD	0.24	0.33	0.30	0.34	0.08	0.67	1.46	1.92
OEF	0.11	0.48	0.66	0.71	-0.02	0.43	1.15	1.43
MIMOSA	0.38	0.54	0.79	0.94	-0.10	0.13	0.79	1.27
SLIM	1.18	0.62	-0.23	-0.18	-0.67	0.95	1.25	0.85
	USA							
DIW/QUEST	4.07	2.38	2.34	2.51	0.05	2.59	4.10	5.00
GEM	1.62	1.23	1.03	0.98	0.52	1.72	2.62	2.98
OECD	1.80	1.20	1.07	0.90	0.83	2.84	5.25	6.40
OEF	2.83	3.55	3.20	3.00	0.01	0.63	1.56	1.92
MIMOSA	4.97	4.58	4.99	5.35	0.22	3.57	9.23	12.94
SLIM	2.43	3.76	2.68	2.52	0.00	1.01	2.23	2.74
	other countries in SLIM							
Belgium	0.54	-0.17	-0.31	-0.12	0.06	0.44	0.19	0.10
Denmark	0.73	-0.13	-0.41	-0.20	-0.15	0.53	0.80	0.61
Ireland	0.51	0.39	-0.40	-0.30	-0.24	0.24	1.00	0.89
Netherlands	0.60	-0.13	-0.40	-0.19	-0.02	0.52	0.57	0.45
Japan	0.37	0.09	-0.17	-0.12	0.06	0.34	0.23	0.09

Figure 4.19: Three fiscal shock experiments

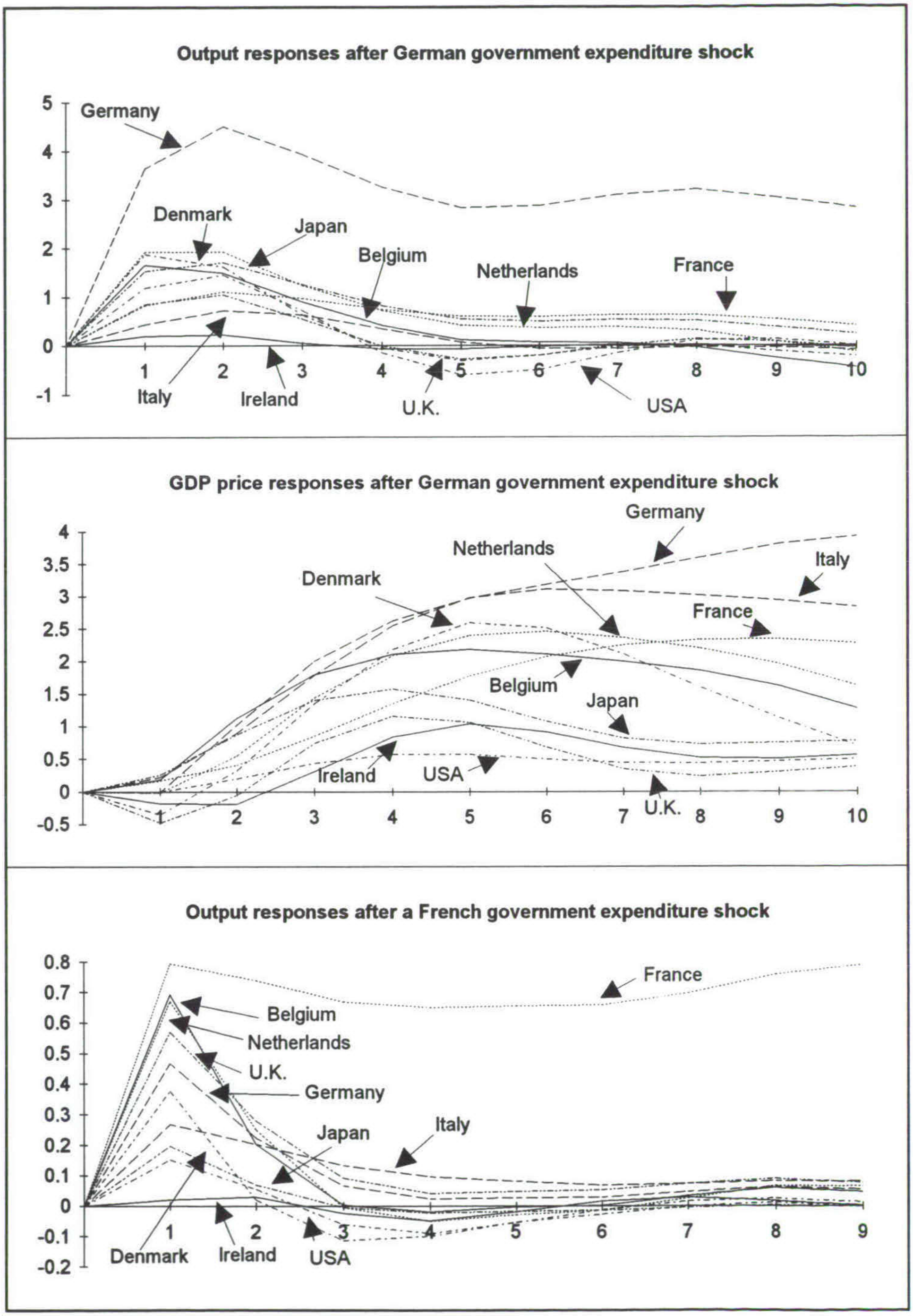
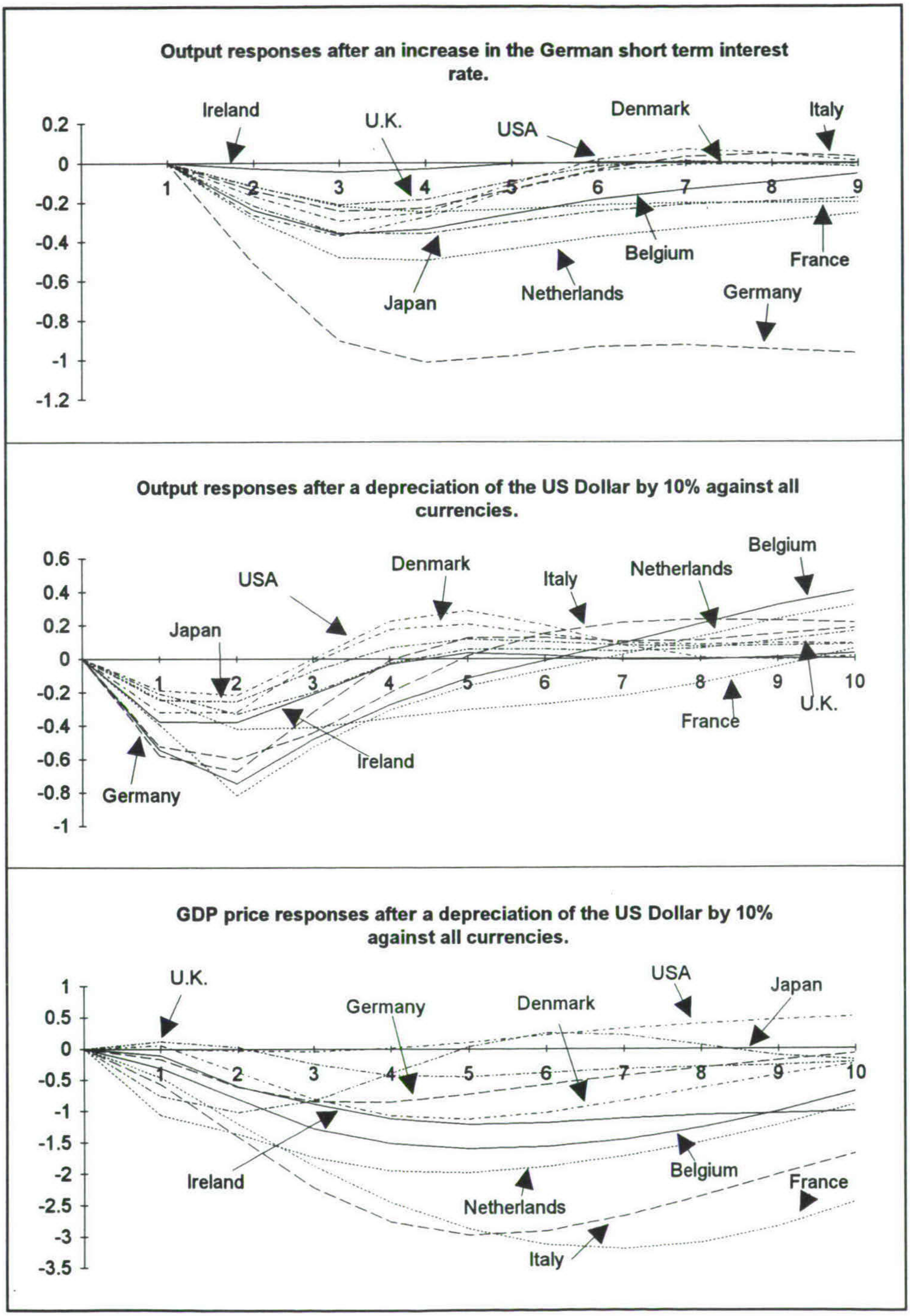


Figure 4.20: Three monetary shock experiments



employment increases for all countries. Because labour force is exogenous the unemployment rate falls with the same amount. We do not observe many movements in the long term interest rates.

(4) Short term interest rate shock Germany.
Note, that in this experiment we do not have fixed real interest rates; so we have an additional feedback transmission mechanism in the aggregate demand equation through prices in the real interest rate term. As a consequence, output responses peter out less quickly than in the previous government expenditure experiments. We see in the first graph of figure 4.20 that in year 2 output responses are negative in all countries and range from 0.06% till 0.55% of German GDP output in the second year. If we exclude Ireland, we see that this effect is still visible in year 5, where the output responses range from 0.09% till 0.45% of German GDP output in year 6. It takes some time before prices respond. In general countries with high output responses show high price responses.

(5) A depreciation of the US dollar by 10% against all other currencies.
By applying this external shock, we expected that an initial depreciation of the US dollar would lower output in the EU-economies as a result of weakened trade competitiveness. Some of this reductioned output might be weakened by the initial expected increase in US demand. As a consequence, prices are expected to fall in the EU-economies. This pattern is clearly visible in the second and third graph of figure 4.20 in the short term. All EU-economies show a negative output and price response. The size of the output responses are moderate and are (in absolute size) never higher than 1%. A major reason for this is the negative response of GDP output in the USA in the first three years, which is due to the direct link in the aggregate demand equation of the USA with Germany. It takes some time before this negative effect is offset and output becomes positive in the USA after year 3. Because of this positive output response in the USA, almost all countries show an output level rise after year 3 and some output responses even get strongly positive in the long-run. In the third graph of figure 4.20 we see that for all the EU-economies prices do fall during the whole period, where Italy and France show the strongest price responses. The direct price link between Japan and the USA in the model evokes the positive price response in the medium term in Japan.

4.6 Conclusions

In this chapter we presented SLIM, a Small Linear Interdependent Model of eight EU-economies, the USA and Japan. The model is of the Mundell-Fleming type and contains six behavioural equations and is estimated with yearly data from 1960 to 1991. The main feature of the model is that direct linkages among countries are explicitly modelled. The model contains international linkages in five of the six equations; namely the equation for the long term interest rate, the two price equations, the GDP equation and the employment equation. The model is designed such that we adopted the same broad specification for the different countries and that the estimation process decides about the strength of certain structures in the model. The same approach is also used for the direct linkages; the estimation process determines the strength of certain linkages. These linkages are modelled such that more emphasis is put on short term effects than on long term effects. The results of the historical tracking performance indicate that our model is capable of reproducing the most important economic developments during the sample period.

Although no stock adjustment and, hence, no integration wealth effects are considered ab initio, the starting point of our interdependent modelling exercise was, as already noted above, a Mundell-Fleming model. The basic model is extended in various ways:
i) more than two countries were included (10 countries in our case), where the countries with the largest trade shares determine the direct linkage specification;
ii) flexible prices, indicating imperfect competition on the goods, labour and capital markets, were incorporated in the equations for output prices, consumer prices, wage rates and long term interest rates;
iii) a labour market part was supplemented, determining a labour demand function, an unemployment function and corresponding prices for labour and output, where the underlying production function is of the Cobb-Douglas type; these equations form the supply part of the model;
iv) finally, a dynamic formulation, allowing for a partial adjustment and an error correction mechanism, was applied.

It is clear that using a Mundell-Fleming type of model as a starting point may be a more appropriate description for some countries than for other countries. Our experience was that the yearly data of small countries like Belgium, the Netherlands and to a lesser extent Denmark *fit* fairly well into this framework. Larger countries like Germany, France and to a lesser extent Italy did also reasonably well. We found major problems for two countries: the United Kingdom and Ireland. For the United Kingdom we had many problems finding suitable aggregate demand, price and wage equations. Our simulation results for Ireland bring on many opposite results as expected from the theoretical Mundell-Fleming model. The models of two outside economies, the USA and Japan, should be treated with more care because we ignored for these countries some important trading partners. Taking this into account the outcomes for the USA were satisfactory whereas for Japan we had large problems finding suitable aggregate demand, price and employment equations.

Through shock analysis our model is compared with five multi-country models as operating

in 1992 at several EU-institutions. With our simple linear model, it is possible to generate (more or less) the same outcomes of some of the main key macroeconomic variables as modelled in large multi-country models. The main differences of our model with the large multi-country models are as follows:
i) An output shock in a country has only little responses for employment in that country, and this effect tends to zero in the long-run. One of the main reasons for finding this effect is that we did not impose any restrictions in the employment equation.
ii) For most countries, we find less inflationary effects from a demand expansion which implies a fairly flat aggregate supply schedule in the short and medium term.
iii) In the fiscal shock experiments we find more differentiation between countries, which is due to the fact that we estimated the effects of government expenditure in the aggregate demand equation.
iv) A global fiscal shock in one of the major EU-economies has (more or less) the same effect on other EU-economies as a fiscal shock originating in the USA. Furthermore, the quantitative size of these shocks are in the short run more substantial than in the long run and range (roughly) in the first year between 0.03%-0.87% GDP output of the country originating the shock.
v) A depreciation of the US Dollar by 10% against all other currencies has only a modest negative effect on output and prices for all EU countries in the short and medium term.

Some country-specific arguments, which appear to be striking in our model, are summarised as follows:
i) An expansive domestic fiscal policy seems to be favourable for the larger economies, the USA, Japan and Germany.
ii) There exist strong long-run price-responses, after applying a wage shock, in the USA and Italy, and weak price-responses in Belgium and Japan.
iii) The effect of a monetary expansion on prices is negative in all countries, except for the price responses in the United Kingdom and Ireland, which are slightly positive in the short run.
iv) The effects of a domestic nominal exchange rate shock on output prices is rather low in the three large economies Germany, the United Kingdom and Japan.
v) The small open economies Belgium, Denmark and the Netherlands profit most from a shock originating in Germany. Belgium, the Netherlands and the United Kingdom profit most from a shock originating in France.
vi) A fiscal expansion in the USA has a large effect on the British and German output and a small effect on the Italian output, but a large effect on Italian prices.

It should be stressed that many of these findings are model dependent. The modelling strategy used, concerning external effects, is that all external influences are measured by growth figures. This may be a somewhat broad measure and covers also external effects which may be caused in the past by factors which are common to many EU-economies. However, we believe that through the strong desaggregation of the large multi-country models, important indirect external effects may disappear. In these models trade volumes are linked through export and import volumes whereas in our model GDP volumes of the various countries are linked. The first method has the disadvantage that it neglects certain effects, such as foreign investments and knowledge. For instance, an invention which stimulates growth in one country can be copied by another country which stimulates growth in that country as well. Such indirect spillover effects are not necessarily captured if one considers only trade volumes. Furthermore, the increasing integration process of the EU-economies makes it likely that strong external effects will become more and more likely in the (near) future which makes it necessary to study models which contain strong (direct) linkages.

The model in its present form can be extended in various directions, such as the inclusion of endogenous exchange rates and intertemporal elements like wealth effects, a government budget constraint and a balance of payments relationship.

Chapter 5

Empirical results

5.1 Introduction

Since the Maastricht Treaty (1991), there is a lot of debate on the agreed convergence conditions. The Treaty sets out four quantified convergence criteria, (1) (low) inflation performance, (2) fiscal consolidation, (3) interest rate stability, and (4) perfect exchange rate stability. For each Member State of the European Union (EU) it is a necessary condition to fulfil these four criteria in order to progress to Stage Three of the EMU by 1999. In order to design a reliable macroeconomic policy for a 'home' country, this country has to take into account influences of as well domestic as foreign economic activity. It is clear that during Stage Two, countries will only consider policies which are sustainable through time, i.e., Member States will only pursue policies which satisfy the four mentioned criteria in nominal values, provided these policies also lead to a reasonable macroeconomic performance in other real variables such as GDP growth, employment or unemployment (see, e.g., Crockett [17]). Since all EU-economies can be considered as open economies, there exist strong interdependent economic relationships among these economies. Furthermore, the increasing integration process will strengthen even more the economic interdependence between countries in the future, which will increase the importance of cross-border effects. Since the criteria indicate that each Member State has to follow the interest rates and the inflation rates set by other countries, it is likely that this 'convergence constraint' also influences economic policymaking of the EU-economies. We want to study the impact on EU-policymaking if Member States are constrained by conditions, in this case the convergence conditions, set out by a supranational authority, in our case the European Commission. We will investigate the economic consequences for the EU-Member States for the short and medium term, i.e. for Stage Two of EMU. It is argued by many authors

that in order to reach Stage Three, various costs are involved (see, e.g., Bean [4], Brandsma and Italianer [9], Buiter [13] and Crockett [17]). For instance, the criteria emphasise a low inflation rate and a sound public finance for the EU-Member States, but it may well be that this hampers real welfare improvements like high GDP-growth or low unemployment in the short and/or medium term. Furthermore there is the argument in the literature, that the loss of freely using the exchange rate as a weapon of macroeconomic management may be costly (see, e.g., Feldstein [28]).
We will study these aspects in a dynamic games context. For example, consider a EU where the Member States fully cooperate. In that case any restriction, thus also the restriction implied by the convergence conditions, will decrease total welfare. In that case it is, therefore, reasonable to search for a measure of the costs of convergence for each country separately and for the EU as a whole. On the other hand, if we assume that within the EU Member States do not fully cooperate, i.e., if we are willing to consider a situation where EU-Member States agree on shared policy targets but do not necessarily cooperate in order to achieve these targets, then it is not straightforward that the imposition of the convergence conditions will always lead to a lower total welfare. In that case it may well be possible that the direction of the spillover effects are influenced in such a way that, through the imposition of the convergence conditions, the size of negative spillover effects decreases. If that is the case, then the convergence restriction may not only be beneficial for total welfare in the EU but it may also be less disadvantageous for each Member State independently in the short to medium term.

In this thesis we model the impact of the convergence conditions as a restriction on each country's policymaking. In a cooperative scenario we consider the full coordination outcome and the outcome where the EU-Member States play cooperatively, but are restricted by the convergence criteria. We will investigate how much convergence is possible/desirable and what are the economic consequences for each Member State. For the noncooperative case we will check whether the imposition of the convergence conditions are profitable or not. As starting point we will return to the Maastricht Treaty (1991) and consider four types of hypothetical scenarios:
(1) Despite of the Maastricht Treaty there is no agreement reached among the EU-Member States: this is represented by a noncooperative scenario (feedback Nash solution).
(2) No agreement reached as in (1), but now each Member State is additionally constrained by the convergence criteria set out in the Maastricht Treaty ('noncooperative convergence solution').
(3) There is an agreement reached about full coordination: this is represented by a purely cooperative scenario (Nash bargaining solution).
(4) An agreement reached as in (3), but there is an additional constraint imposed by the convergence criteria ('cooperative convergence solution').

In this chapter we compute these four (extreme) scenarios for the SLIM-model as described in the previous chapter.

In the literature, there is a lot of intellectual debate about the consequences of the convergence conditions (see, e.g., Bean [4], Crockett [17] and Eichengreen [25]). However, an empirical dynamic game analysis of the consequences of the convergence conditions for the EU-economies during Stage Two is not made before. We are aware of one simulation study by Brandsma and Italianer [9] using the European Commission's Quest model. In that study the convergence criteria are considered as an example of agreement on shared policy targets in the sense that the desired paths of the individual economies are jointly finetuned and determined. They argue (see page 12): 'Since it is difficult to shoot at a moving target, the potentially best performers should make their targets explicit and also make it clear that they will not try to push inflation much below that point of reference, even when inflationary pressures are weakening abroad'. Therefore, the points of reference for the inflation rate and the long term interest rate are fixed beforehand in their analysis. It may be clear that, in a dynamic context, the reference points may change during Stage Two. For instance, if there is an upward swing of GDP-growth in all EU-countries it is likely that inflation rates, and, hence, also the reference points, will increase too. In that respect, the empirical analyses in this paper can be seen as an extension of the study of Brandsma and Italianer [9], since, by constructing a convergence function, we take the possibility of a moving target into account. As we will show, the distinction between a convergence function and individual welfare functions yields various additional advantages since we are now able to study the following aspects:
(1) How will the reference points of the nominal long term interest rate and the consumer price inflation evolve over time?
(2) How much convergence is possible/desirable and how will the convergence aspects affect welfare in the short and medium run as well for real as for nominal variables in each Member State?
(3) Which Member States are able to fulfil the criteria, and which not?
(4) Are the convergence criteria, as set out in the Maastricht Treaty, reasonably specified or are (minor) revisions necessary? [1]

In section 5.2 we will first recall some key properties of the SLIM-model. In section 5.3 we will present a description about the four possible game outcomes where we will put most emphasis on the convergence solutions, since the ideas behind this concept are new in the international policy coordination literature. In section 5.4 we will start our empirical

[1] The empirical results of Brandsma and Italianer [9] imply that if the Member States play a purely cooperative strategy, almost all EU-countries will be able to fulfil the criteria. It is, however, important to note that in this research the authors use the wage rate as instrumental variable and they allow for dismissing (government) employees.

work and present objective functions for each country and a convergence function. We will specify desired paths and penalty weights for these functions. In section 5.5 we will present and analyse the outcomes of our experiment and in section 5.6 we will conclude.

5.2 The SLIM-model revisited

As noted in the introduction we use the SLIM-model to perform our dynamic game analyses. For an extensive description of the model we refer to chapter four; here we will briefly recall some key properties of the model. In the SLIM-model, each country is represented by six behavioural equations which contain strong interactions. In table 5.1 we

	Ge	Fr	UK	It	Nl	Be	Dn	Ir	USA	Japan
Ge		•	•						•	•
Fr	•		•						•	
UK	•	•							•	
It	•	•							•	
Nl	•	•	•			•			•	
Be	•	•			•					
Dn	•		•						•	
Ir	•		•						•	
USA	•									•
Japan	•								•	

Table 5.1: Direct Interdependencies among the countries in the SLIM-model

summarise how these interdependencies are modelled. For example the dot in the upper row of Germany belonging to the column of France indicates that Germany directly depends on France. Hence, for the behavioural equations published in chapter four, this means that for at least one equation of Germany a French economic variable enters directly that equation. Comparing this table with table 4.4 in chapter four we see that almost all countries considered as most important trading partners in table 4.4 also appear in this table 5.1. In our estimation procedure we only did not find any direct impact for Italy on Germany and France. Thus, in the SLIM-model the EU-countries such as France, Germany and the United Kingdom are strongly mutually interdependent, whereas open economies as Belgium, Denmark, Ireland, Italy and the Netherlands are unilaterally affected by the larger EU-economies (i.e., there is no feedback transmission from these EU-economies to the larger three economies). The model is also designed such that all EU-economies are directly (or indirectly) interdependent with the USA and Japan. To illustrate the behaviour of the model we briefly state in table 5.2 the general form of the six behavioural equations. Equation (1) expresses that real output, Y, depends positively on the real exchange rate,

Table 5.2: Equations in the SLIM-model for one country[a]

Equation number	*Equation*
(1)	$Y = f(E + P_y^* - P_y, RL - \Delta P_y, Y^*, G)$
(2)	$P_y = f(W, E + P_y^*, Y - \bar{Y})$
(3)	$P_c = f(P_y, E + P_y^*)$
(4)	$N = f(W - P_y, Y, E + P_y^* - P_y)$
(5)	$W = f(P_c, \Delta(L - N), Y - N)$
(6)	$RL = f(RL^*, RS, \Delta P_c)$

a. All variables are in logaritmic form, except RS, RL and $(L - N)$ which are in rates. Δ indicates 'first differences' and the superscript * denotes foreign variables.

$E + P_y^* - P_y$, negatively on the real long term interest rate, $RL - \Delta P_y$, positively on foreign output, Y^*, and real government expenditure G. In equation (2) the output price, P_y, depends positively on nominal wages, W, import prices, $E + P_y^*$, and deviations from trend output, $Y - \bar{Y}$. In equation (3) consumer prices, P_c, depend positively on domestic output prices and foreign output prices $E + P_y^*$. Labour demand in equation (4) depends negatively upon real wages, $W - P_y$, positively on real output and on the gap between foreign and domestic output prices, $E + P_y^* - P_y$. The impact of the gap between foreign and domestic output prices in this labour demand equation is ambiguous and depends upon the country under consideration (see chapter four). Nominal wages in equation (5) depend positively on consumer prices, negatively on the change in unemployment, $\Delta(L - N)$, and positively on labour productivity, $Y - N$. The nominal long term interest rate, RL, in equation (6) depends positively on the foreign nominal long term interest rate, RL^*, and the domestic nominal short term interest rate, RS, and consumer price inflation, ΔP_c.

Since unemployment is defined as the labour force minus employment, the model contains, for each country, four exogenous variables, G, government expenditure, RS, the short term interest rate, E, the nominal exchange rate, and L, the labour force.

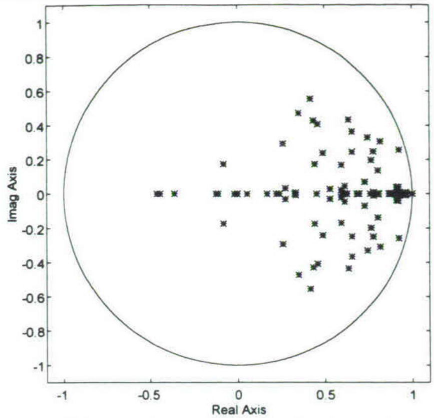


Figure 5.1: Eigenvalues of A in the imaginary plane.

After estimation the SLIM model can be described by the following set of equations:

$$y_t = \hat{A}_0 y_t + \hat{A}_1 y_{t-1} + \hat{A}_2 y_{t-2} + \hat{B}_0 u_t + \hat{B}_1 u_{t-1} + \hat{B}_2 u_{t-2} + \hat{D}_0 ex_t + \hat{D}_1 ex_{t-1} + \hat{D}_2 ex_{t-2} + \hat{\varepsilon}_t$$

where $\hat{A}_0, \hat{A}_1, \hat{A}_2, \hat{B}_0, \hat{B}_1, \hat{B}_2, \hat{D}_0, \hat{D}_1$ and $\hat{D}_2$ contain the estimated coefficients and y represents the vector of endogenous variables, u_t the vector of instrumental variables, ex_t the vector of exogenous variables and $\hat{\varepsilon}_t$ the vector of errors in the model. Next, these equations are transformed into state-space form (see, e.g., de Zeeuw [85]). This yields the following set of equations:

$$\begin{aligned} x_{t+1} &= \hat{A}x_t + \hat{B}\bar{u}_t + z_t \\ \bar{y}_t &= \hat{C}x_t. \end{aligned}$$

In this standard discrete time state-space form, x_t, represents the vector of state variables involving endogenous and instrumental variables, $\bar{y}_t$, represents the vector of objective variables for all policymakers, $\bar{u}_t$, the vector of instrumental variables for all policymakers and z_t the vector of exogenous variables at time t, $\hat{C}$ is specified such that the objectives in $\bar{y}_t$ are, for each country, GDP-growth, GDP-inflation, consumer price inflation, growth in wages, growth in employment and the nominal long term interest rate. In this study we consider two sets of instrumental (policy) variables for $\bar{u}_t$. In our first dynamic game experiment we just consider the nominal short term interest rate and government expenditure as instrumental variables, whereas in the second experiment we also include the exchange rate as instrumental (policy) variable. To get an idea of the size of the model, $\hat{A}$ is a 180x180 matrix, $\hat{B}$ is a 20x180 matrix or 30x180 matrix depending on the set of instrumental variables used and $\hat{C}$ is a 60x180 matrix. In a discrete time-model, as used here, the stability properties of the model can be easily checked by an eigenvalue plot. This plot is shown in figure 5.1, where the eigenvalues of $\hat{A}$ and the unit circle are plotted. If the eigenvalues of $\hat{A}$ lie inside the unit circle then the model is said to be stable. As we can see, some of the eigenvalues fall on the unit circle in (1,0), and hence at least one equation contains a unit root. These equations are, e.g., price equations. These equations contain a unit root in their level specifications but are stable in their first differences. This is, however, not such a problem since in our dynamic game analyses we are mainly interested in the first differences (growth values) of the various variables.

5.3 Description of the dynamic game

It is common use to compare noncooperative and cooperative outcomes when applying dynamic game theory in economics. However, empirical studies with large scale models appear less often. One of the first well-known empirical studies is the Oudiz and Sachs paper [61]; more recent studies are Ghosh and Masson [34], Hughes Hallett [39] and McKibbin and Sachs [55]. In this paper we compare two generally known dynamic game equilibrium concepts, the noncooperative feedback Nash solution and a cooperative Pareto solution, represented by the axiomatic Nash bargaining outcome, with two other new concepts in which the players are restricted in their policy choice. This restriction is imposed by the convergence conditions, which can be interpreted as a dynamic constraint on the (non)cooperative game. We will now briefly explain the four solution concepts. Consider again the Maastricht Treaty (1991). The scheme in figure 5.2 describes the situation of

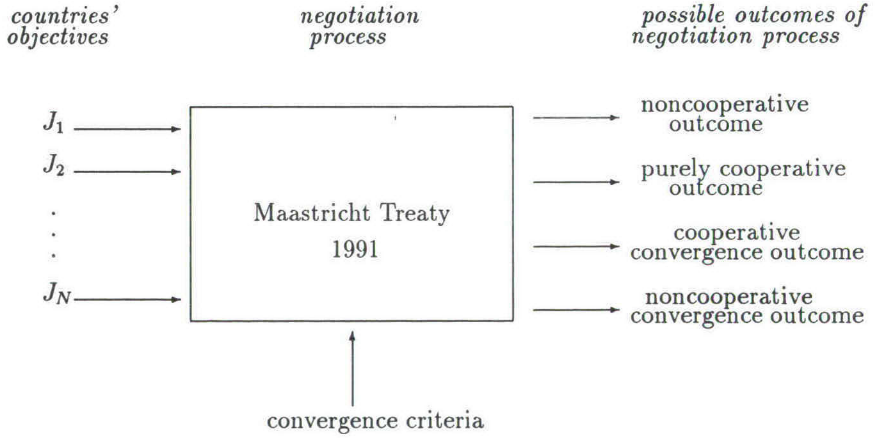


Figure 5.2: Scheme of the negotiation process of the Maastricht Treaty.

the EU-Member States at the time of the Treaty. Each Member State enters the negotiation process with a certain target or objective function, represented by $J_i, i = 1, ..., N$. As is common practice in dynamic game theory, we assume that this function can be approximated well by a quadratic functional[2]:

$$J_i = \sum_{t=1}^{T} y_i^A(t)' Q_i(t) y_i^A(t) + u_i^A(t)' R_i(t) u_i^A(t), \tag{5.1}$$

where $y_i^A(t) := y_i(t) - y_i^d(t)$ are the deviations of the values of the target variables from their desired values for each EU-Member State i at time t. Similarly, $u_i^A(t) := u_i(t) - u_i^d(t)$ is the vector of deviations of country i's instruments from their trend (desired) values. Furthermore, the matrix Q_i is assumed to be positive semi-definite and the matrix R_i positive definite for $i = 1, ..., N$. Now, each country $i, i = 1, ..., N$, enters the negotiation process with its objective J_i which has to be minimised subject to a set of linear(ised) dynamic constraints, represented by the SLIM-model as explained in section two. Furthermore, we assume that the European Commission enters the negotiation process with as objective criterion a quadratic functional on the convergence conditions of the Maastricht Treaty. It is clear that we are now entering the field of dynamic game theory since each country's behaviour is dependent on the other countries' behaviour. This can, of course, lead to many possible outcomes; therefore, we will restrict ourselves to four (extreme) cases.

5.3.1 case 1: Noncooperative policy

Since there are no clearly established 'rules of the game', we assume an environment where each Member State has complete information and acts individually rationally. For an appropriate description of a noncooperative game outcome we use the feedback Nash equilibrium. The feedback Nash equilibrium has some more desirable properties than other Nash equilibria, such as strong time consistency and stochastic robustness (see, e.g., Basar and Olsder [3], de Zeeuw and van der Ploeg [86] and Holly and Hughes Hallett [44] for a comparison of different Nash equilibria and for the mathematical expressions for computing the unique feedback Nash outcome). Furthermore, the feedback Nash equilibrium is generically unique in a linear quadratic framework with a finite planning horizon (see Basar and Olsder [3]). We will follow a similar strategy here and use the feedback Nash equilibrium as the threatpoint of the game. Thus, we assume that the EU-Member States play a noncooperative game without taking the convergence conditions into consideration. Remark that this is perfectly acceptable in the two player case, but, since in our case there are more players, we additionally assume that there will be no coalitions among the players

[2] Advantages and limitations of quadratic loss functions are discussed, a.o., in Petit [66]

[3]. So, here we assume that there is no cooperation among countries at all. In the sequel we will denote this outcome by *NC* and the welfare outcome of this noncooperative solution by $J^{NC} := \{J_1^{NC}, ..., J_N^{NC}\}$, where N represents the number of players (countries).

5.3.2 case 2: Cooperative policy

For the purely cooperative case we assume that the countries agree on the Nash bargaining outcome which is a Pareto optimal outcome (see, e.g., Nash [58]). It is well-known that this solution can be obtained by minimising the 'collective' loss function:

$$J = \sum_{i=1}^{N} \alpha_i J_i, \quad \text{with} \quad \alpha_i \geq 0, \ \sum_{i=1}^{N} \alpha_i = 1, \tag{5.2}$$

subject to the linear constraints represented by the SLIM-model, and in which the set $\{\alpha_1, ..., \alpha_N\}$ is chosen as $\alpha^{NB} := \{\alpha_1^{NB}, ..., \alpha_N^{NB}\}$ which corresponds to the Nash bargaining solution. This solution, which we will denote by *NB*, has some desirable properties. For instance, there exists a unique relationship between the 'welfare weights' α_i^{NB}, for $i = 1, ..., N$, the disagreement point, represented by the noncooperative solution J^{NC} in case 1 and the welfare outcome of the Nash bargaining solution, say $J^{NB} = \{J_1^{NB}, ..., J_N^{NB}\}$[4]:

$$\alpha_i^{NB} = \frac{\prod_{i \neq j}(J_i^{NC} - J_i^{NB})}{\sum_{i=1}^{N} \prod_{i \neq j}(J_i^{NC} - J_i^{NB})}$$

This relationship implies that:

$$\alpha_1^{NB}(J_1^{NC} - J_1^{NB}) = \alpha_2^{NB}(J_2^{NC} - J_2^{NB}) = \cdots = \alpha_N^{NB}(J_N^{NC} - J_N^{NB}). \tag{5.3}$$

Since the deviations in the welfare functions are all described in percentage points, we may assume that the welfare functions, J_i, are (roughly) comparable among countries, i.e., if $J_i > J_j$, then we can argue that policymaking for country i is more costly than for country j. This observation makes it possible to interprete relationship (5.3) as follows (see also chapter 3): a player who gains more from playing cooperatively is more willing to accept a smaller 'welfare weight' than the player(s) who gain(s) less. Alternatively, a player who gains less may demand a higher 'welfare weight' by threatening not to coordinate, knowing that the potential loss from no agreement is larger for the other player(s). We will use this argument for interpreting some of our results.

The two previous cases are standard in dynamic economic game theory. In the following

[3]Of course, it is possible to take this coalition aspect into account. However, regarding the fact that the number of coalitions between 8 EU-countries is already very large, we use here this simplifying assumption.

[4]For a proof of this relationship we refer to chapter 3.

two subsections we will elaborate the concept which deals with the impact of the European Commission. In this case we assume that the European Commission, as an independent negotiation partner, is involved in the negotiation process as well. We assume that the European Commission has its own objectives, i.e., the convergence criteria as specified by the Maastricht Treaty, which can be quantified in a convergence function. We will denote this convergence function by C and assume that C is quadratic (like the individual objective functionals $J_i, i = 1, ..., N$). The general form of this function is

$$C = \sum_{i=1}^{N} C_i, \quad \text{with} \quad C_i = \sum_{t=1}^{T} y_i^C(t)' Q_i^C(t) y_i^C(t), \tag{5.4}$$

where $y_i^C(t) := y_i(t) - y^c(t)$ represents, for each country, the deviation of its target vector from a reference vector at period t. Note that this 'reference vector' $y^c(t)$ will not be fixed beforehand for the complete planning period, but should at each time period t be considered as a function of the target vectors $y_1(t), ..., y_N(t)$ and will be determined within the optimisation procedure itself; C is defined as the sum of the individual countries' convergence functions C_i. In practice it is possible that each country has its own reference vector in mind to which it wants to converge in order to reach the convergence criteria but, for practical reasons, we assume that countries cooperatively agree on the same reference vector $y^c(t)$. The time dependent weight matrices $Q_i^c(t)$ contain the relative priorities which each individual country wants to assign to certain convergence aspects. We refer to Section 4 for these subjects and for a precise formulation of the convergence function and do not elaborate these subjects further here. In that section we will also discuss the fact that there may be some overlap between the countries' own objectives and the convergence objective. In the sequel we first discuss the two convergence game outcomes.

5.3.3 case 3: Cooperative convergence policy

The cooperative convergence outcome is modelled as a restricted cooperative outcome, where the restriction is modelled with the convergence function C. Furthermore, we assume that if one of the EU-Member States does not agree, none of the EU-Member States will agree to play in a cooperative mode, in which case 1 is the appropriate model formulation, i.e., we use NC of case 1 as the threatpoint of the game. Another way to look at the cooperative convergence outcome is that we are dealing with a game between $N+1$ players, where the N countries altogether agree with the convergence criteria as specified by the $(N+1)$st player. One could argue that this $(N+1)$st player, represented here by the European Commission, has the power to conduct the coordination process between the N Member States and that a possible withdraw of one of the Member States from the negotiation process would be the starting point of a breaking down of the European Union, i.e., will lead to case 1. Now we argue that there are not many incentives for one of the EU-Member States to disagree with this convergence condition as long as their corresponding individual costs $J_i, i = 1, ..., N$, will be lower than the costs which are represented by the noncooperative solution. Since all the EU-Member States are interested in the convergence aspect (in order to reach Stage Three which involves the creation of a full monetary union by 1999) we assume that they will try to converge as much as possible as long as each individual EU-Member State is better off compared to the noncooperative case. As an

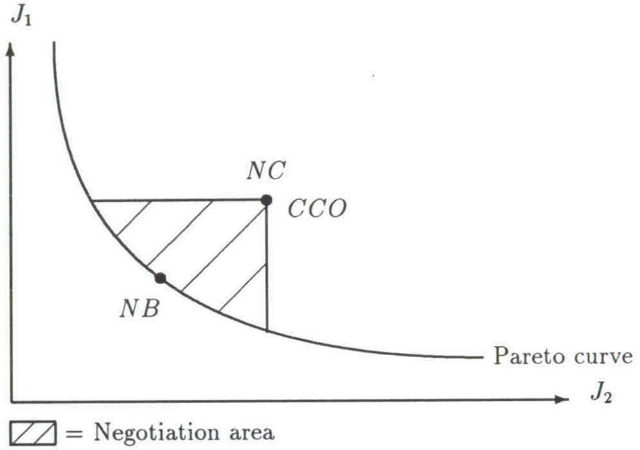


Figure 5.3: The negotiation area in the J_1, J_2-plane for the two player case.

example consider the two player case. The picture in figure 5.3 represents the J_1, J_2 plane. The convex Pareto curve represents all possible cooperative solutions. The Nash bargaining solution is denoted by NB and represents case 2. The outcome denoted by NC is the threatpoint and represents case 1. Remark now that all possible game outcomes outside the shaded 'negotiation area' are not interesting for at least one of the players since, then, at least one is better off in case 1. In case 3 the EU-Member States are restricted in their policy choice by this 'negotiation area'. The maximum convergence that can be reached is a cooperative policy which stays inside the 'negotiation area' and maximises convergence (i.e., minimises C). We denoted this outcome in the figure with CCO, which represents case 3. From the results in chapter two, we have that, in the N-dimensional case, all possible cooperative convergence outcomes can be calculated by minimising an augmented 'collective' loss function:

$$J^C = (1-\lambda)\sum_{i=1}^{N}\alpha_i J_i + \lambda C, \quad \text{with} \quad \sum_{i=1}^{N}\alpha_i = 1, \;\; 0 \le \alpha_i, \lambda \le 1, \tag{5.5}$$

subject to a set of constraints represented by the SLIM-model in our case. Each cooperative convergence outcome can be represented by a particular set of $\{\alpha_1, ..., \alpha_N, \lambda\}$. In the sequel we will represent the CCO outcome, which maximises convergence within the 'negotiation area', by $\alpha^{CCO} := \{\alpha_1^{CCO}, ..., \alpha_N^{CCO}, \lambda^{CCO}\}$. In chapter two we also proved that this CCO outcome is uniquely determined, and that this outcome coincides in the $J_1, ..., J_N$ plane with the noncooperative outcome NC. Therefore, the dot in figure 5.3, which lies on the corner of the 'negotiation area', represents both; the NC and the CCO outcome. Note, however, that the belonging policy choices of both outcomes generally differ. In Appendix A we describe the formulae and the (constrained) numerical optimisation algorithm for finding this cooperative convergence solution CCO.

5.3.4 case 4: Noncooperative convergence policy

In this subsection we assume that the EU-Member States pursue a noncooperative policy which is restricted by the convergence criteria. We assume that each Member State i minimises $\tilde{J}_i$, with

$$\tilde{J}_i = (1-\lambda_i)J_i + \lambda_i C_i \quad \text{with} \quad C_i \text{ as in (5.4)}, 0 \le \lambda_i \le 1$$

and where λ_i is the relative weight each player assigns to his own convergence. For simplicity, we assume that all players choose the same value for $\lambda_i := \lambda$ and we assume that there is an agreement that the Member States shoot on the same moving target $y^c(t)$. The main difference with case 3 is that each individual country, say country i, tries to minimise its part, C_i, of the total convergence function C in a noncooperative game, instead of minimising C cooperatively. Beforehand, it is hard to say whether this 'cooperative agreement' on $y^c(t)$ influences the individual welfare functions J_i, in comparison to the NC outcome of case 1, positively or negatively. Furthermore, it is interesting to see whether this agreement on the same moving target $y^c(t)$ really will lead to convergence in a noncooperative world. In our empirical application we will actually investigate whether this outcome falls inside or outside the negotiation area as specified in figure 3. In the sequel, we will represent this noncooperative convergence outcome by NCO and we willl denote its welfare costs by $J^{NCO} := \{J_1^{NCO}, ..., J_N^{NCO}\}$. Remark that we are interested in comparisons of different outcomes in the $J_1, ..., J_N$-plane and we specify the NCO outcome accordingly [5]. To understand the impact of a constraint on a dynamic game more in general we discuss

[5]Another viewpoint would be to scale everything into the $\tilde{J}_1, ..., \tilde{J}_N$-plane, but this yields the additional problem of the determination of λ_i.

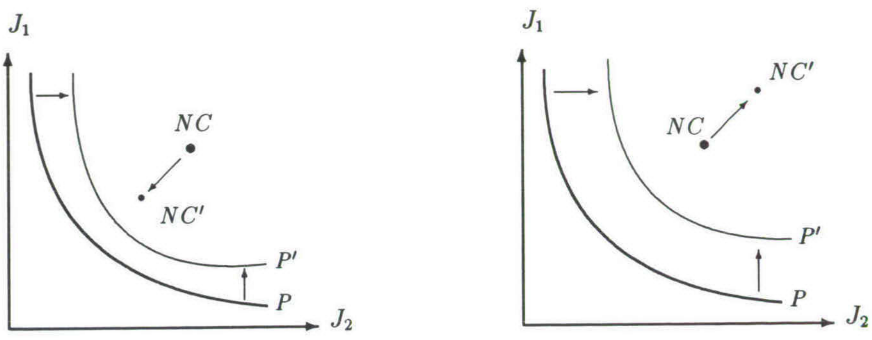


Figure 5.4: Two examples of effects on individual welfare-loss originating from a restriction.

graphically, in the two player case, what kind of properties an 'ideal' constraint (imposed by some kind of central authority) should posses. Remark, that in the following idea the authority, which imposes the restriction, does not know beforehand what kind of game the two players play. Consider again the Pareto curve P and the threatpoint NC in both diagrams of figure 5.4 and consider a possible restriction on the game. Since any restriction in the cooperative game leads to welfare loss in the J_1, J_2-plane, we observe that in the cooperative restricted game the Pareto frontier P (in the J_1, J_2-plane) moves to the north-east direction. In the noncooperative game it is, generally, unknown in which direction the NC point moves. It could move in any direction. As an example consider the two diagrams in figure 5.4. In general, the first diagram is an example of an 'ideal' restriction; the restricted Pareto frontier,P', lies close to the Pareto frontier, P, and the restricted noncooperative point, NC', moves to the south-west direction of the NC-outcome. An example of a 'bad' restriction is visible in the second diagram of figure 5.4; the NC threatpoint and the Pareto frontier move substantially to the north-west direction.

5.4 Specification of objectives and priorities

Since the dynamic game calculations with the SLIM-model are only relevant for the EU-Member States, we exogenise USA and Japan in the model. In Appendix C.2 we will describe the exogenous choices for these countries' exogenous variables and for the other exogenous variables in the model, such as the labour force. We will use for each country

the nominal short term interest rate and the level of government expenditure as policy variables. For the other exogenous variable, the nominal exchange rate, the Maastricht Treaty imposes perfect exchange rate stability at Stage Three. Therefore, we adopt two approaches in this chapter.
(1) Firstly, we fix the dollar exchange rate on the 1991 level for the complete planning period 1992-1999; in this case we assume a fixed (dollar) exchange rate regime.
(2) Secondly, we use the exchange rate as a policy variable wich receives a very high weight in the welfare function. In this case we allow for small movements of the exchange rate around the desired paths. Since in the SLIM-model each exchange rate within the EMS is modelled through the dollar we consider small movements around the ideal paths of their currency against the dollar. Hence, we have also tight exchange rates within the EMS.
The motivation behind these two approaches is that we want to investigate the possible gains from exchange rate management as well in a cooperative as in a noncooperative world.
It is clear that in practice the monetary authorities do not have the power to 'fully' control the exchange rate. Therefore, study (2) be interpreted as a sensitivity analysis to dynamic game study (1) and should give more insight into a question like: Is it possible to improve convergence considerably if we allow for small movements in the exchange rate?[6] For instance, the effect of exchange rate managenent in the noncooperative case is not clear, since it may well be the case that a noncooperative use of exchange rate management may lead to lower global and individual welfare for all Member States than in a situation where exchange rates are fixed. Remark, that in this research, we do not consider the possibility that monetary or fiscal policy could create tensions within the exchange rate system [7].

The choice of the desired paths $y_i^d(t)$ and $u_i^d(t)$ is mainly in line with previous studies (see, e.g., Hughes Hallett [38, 39]). We target growth values for Y, P_y, P_c, W, N, nominal level values for RL, RS and real level values for G. The ideal target values for the growth rates are constructed such that we start for each variable with a desired growth rate for 1992. These desired growth rates for 1992 were chosen such that they are in accordance with the actual growth rates of 1991. Then, we constructed linear growth paths towards the ideal growth rates for 1999. For the construction of the ideal paths of the level values we used, in most cases, the true 1991 values (also representing the end of the estimation period in the SLIM-model) as starting values and applied linear interpolation towards the 1999 desired values. In subsection 5.4.1 we describe an individual objective function of each Member State and in subsection 5.4.2 we specify our choice for the convergence function.

[6] Other arguments for using the exchange rate as instrumental variable are given by Petit [66]
[7] The case where the exchange rate is endogenous is a subject for future research.

5.4.1 Individual objective functions and relative priorities

The (growth) values for the ideal paths and their relative priorities are given in table 5.3, presenting each country's desired values for 1992 and 1999 respectively. The desired values for the years inbetween are constructed by linear interpolation. Since we target level variables for real government expenditure, we specify for all countries the levels for the year 1991. We assume that each country, except Ireland, will aim for real output growth of 4%

Table 5.3: The objective function specification for the years 1992-1999. [a]

	Countries							
	Belg.	Denm.	Germ.	France	Irel.	Italy	Netherl.	U.K.
The desired values y_i^d for 1992 and 1999.								
ΔY	2.2-4.0	1.6-4.0	3.7-4.0	1.1-4.0	2.5-5.0	1.6-4.0	2.4-4.0	1.0-4.0
ΔP_y	2.5-2.0	2.4-2.0	3.8-2.0	2.8-2.0	1.5-2.0	6.6-3.0	2.8-2.0	5.8-2.0
ΔP_c	2.6-2.0	2.4-2.0	3.5-2.0	2.9-2.0	3.0-2.0	6.1-3.0	3.2-2.0	6.3-2.0
ΔW	6.0-5.0	3.9-5.0	4.6-5.0	4.5-5.0	5.7-6.0	8.2-7.0	4.4-5.0	7.9-7.0
ΔN	0.3-1.5	0.0-1.5	2.4-1.5	0.3-1.5	0.0-1.5	0.9-1.5	1.3-1.5	0.0-1.5
RL	9.0-7.0	9.1-7.5	8.2-6.0	9.2-7.5	8.9-7.0	12.6-8.0	8.4-6.0	9.8-7.5
The desired values u_i^d for 1992 and 1999.								
RS	8.9-6.0	9.5-7.0	8.7-5.0	9.3-7.0	9.5-7.0	11.4-7.0	8.8-5.0	10.9-8.0
ΔE	0.0-0.0	0.0-0.0	0.0-0.0	0.0-0.0	0.0-0.0	0.0-0.0	0.0-0.0	0.0-0.0
ΔG	1.6-0.0	2.1-1.0	2.4-1.0	2.2-1.0	1.6-1.0	1.4-0.0	3.7-1.0	1.7-1.0
G_{1991}	15.073	14.076	13.073	15.043	12.360	9.340	13.545	12.611

The relative priorities (equal for each country)

Matrix Q						*Matrix R*		
ΔY	ΔP_y	ΔP_c	ΔW	ΔN	RL	RS	G	E (*if included*)
2.0	0.5	1.0	0.5	2.0	0.5	2.0	2.0	10.0

[a]. All units in percentage changes per annum, except G_{1991} in logarithms.

and that Ireland aims for 5% by 1999. Each country strives for a low GDP- and consumer price inflation rate for which we assume 2% as ideal in most countries in 1999 (only Italy 3%). Due to the trade off between, on the one hand the inflation cost component and on the other hand the income component for the improvement/preservation of purchasing power we target growth in nominal wages around 5-7% for the year 1999. the year 1999. Due to the above mentioned trade off the relative priority of nominal wages is assumed to be low (Q value of 0.5). Growth in employment, one of the main concerns of policymakers

today, is given a relatively high priority of 2 and a desired growth path of 1.5% in 1999. Since at the time of the Maastricht Treaty in 1991 the nominal long term interest rates were rather high in each country, we considered a decline of at least 2% during the period 1992-1999 for this variable. Note that at the Maastricht Treaty it is imposed that each Member State should strive for fiscal consolidation (before the end of this century, each country should have a public budget deficit of less than 3% of GDP and its governmental debt should not exceed 60 % of GDP). Therefore, we assume that some countries restrict their level of government expenditure substantially, e.g., for Belgium and Italy we assume a zero percentage growth of the desired government expenditure in 1999. For the other countries we assume an ideal growth path of 1% in 1999.
The priorities policymakers attach to the different variables are reflected in the weights Q and R of the individual objective functions. They are presented at the bottom of table 5.3. We assume that these weights are the same for each Member State and are constant over time[8]. For the desired paths y_i^d we give a relatively high priority to GDP growth and employment growth and relatively low priorities for the growth of nominal wages and the level of the long term interest rate. Concerning the inflation rates, we give a higher priority to the consumer price deflator than to the GDP-price deflator. For the desired paths u_i^d we choose a priority of 2, except for the case where the exchange rate is used as a policy variable. In that situation we give a very high priority of 10 to the exchange rate (and the other policy variables a value of 2), indicating that strong movements in the dollar exchange rates are heavily penalised.

One can decompose the convergence criteria in two types of conditions (see Siebrand [71]). On the one hand conditions which are conducted by the European Commission called *central* conditions and on the other hand *decentral* conditions which are executed by each Member State individually. Examples of centralised policy behaviour are price stabilisation and interest rate stabilisation; budgetary policy is an example of decentralised behaviour. Since a restrictive budgetary policy seems to be sensible anyway, Maastricht Treaty or not, we decided to put this decentral criterion in the individual objective functions and the central criteria in the convergence function. The other distinction which can be made here is that the central conditions are an example of shared *flexible* policy targets, whereas the decentral conditions are an example of *fixed* policy targets. For instance, the reference values for the individual budgets are fixed beforehand, whereas the reference points for the long term interest rates and the inflation rates may fluctuate over time. This decision implies that we are comparing four dynamic game outcomes, all with a 'cooperative agreement' on fixed shared targets, which can be divided in two game outcomes (a noncooperative and a cooperative one) and two other game outcomes (a noncooperative and a cooperative one),

[8]This is a simplification. In practice the relative priorities (and also the desired paths) are (frequently) adjusted by the government.

where the Member States additionally give some priority to the two central criteria which are modelled as flexible shared targets.

5.4.2 The convergence function

As convergence function we propose the following specification:

$$C = \sum_{i=1}^{8} C_i, \text{ with } C_i := \sum_{t=1992}^{t=1999} \delta^t \{(RL^i(t) - \bar{RL}(t))^2 + (\Delta P_c^i(t) - \Delta \bar{P_c}(t))^2\}, \qquad (5.6)$$

where $RL^i(t)$ and $\Delta P_c^i(t)$ are the long term interest rate and the consumer price deflator, in year t for country i, respectively; the bar values represent averages of that year. We make the following choice for these averages:

$$\bar{RL}(t) = \frac{1}{3}\{RL^{Ge}(t) + RL^{Fr}(t) + RL^{UK}(t)\},$$
$$\Delta \bar{P_c}(t) = \frac{1}{3}\{\Delta P^{Ge}(t) + \Delta P^{Fr}(t) + \Delta P^{UK}(t)\},$$

for each year t =1992,...,1999. By taking this particular choice we assume that during the planning period all EU-Member States follow the average level of the nominal long term interest rates and the average consumer price inflation values of the three largest Member States, Germany, France and the United Kingdom. Since the European Commission, (the $(N+1)$st player) acts as a representative of the N-players, we assume that the choice of this particular convergence towards the average of the 'large 3' represents the agreement between all EU-Member States about convergence in consumer price inflation and nominal long term interest rates[9]. The parameter δ_i represents the time preference and/or a convergence weight for each country i. It seems reasonable to assume that convergence becomes more desirable at the end of the planning period; therefore countries will put a higher weight on convergence towards the end of the planning period. We simplicity we assume a constant $\delta = 1.2$ which reflects the fact that the priority for convergence increases by 20% for each country each following year. Remark that the final policy choices will be sensitive for the chosen convergence function and note, furthermore, that the desired values $\bar{RL}(t)$ and $\Delta \bar{P_c}(t)$ for each year t, t =1992,...,1999, are not specified beforehand but will be determined by the optimization procedure, and, hence, by the convergence function itself.

[9]Remark that we have to make this simplifying assumption, since constructing a convergence function which accurately represents the convergence conditions of the Maastricht Treaty is very difficult. This aspect, however, makes the dynamic game analyses not less interesting, since, in this case, it is possible to look for various convergence functions, which in fact could yield conditions which would be quite different from those specified at the Maastricht Treaty. To go even a step further, one could think of the possibility of constructing an 'optimal' convergence function. We refer to Appendix C for other specifications of the convergence function.

5.5 Empirical Results

In this section we will describe the empirical results. In the first subsection we will show the results for the fixed echange rate regime. In the next subsection we perform the same experiment but consider the case where (slight) adjustments in the dollar exchange rate are possible. Exchange rates are kept very tight which is modelled through the high weight in the R-matrix. Therefore, if a particular country tries to adjust its dollar exchange rate it will be heavily penalised. In both dynamic game experiments we assume for the *NCO*-outcome $\lambda = 0.2$, indicating that each country gives a weight of 20% to minimise C_i and a weight of 80% to minimise J_i. We will study for this noncooperative outcome the sensitivity of our results related to the choice of λ and report some results we obtained with different choices for λ.

5.5.1 Empirical results in a fixed exchange rate regime

Table 5.4 contains, for each country, the objective function values and the convergence values for each game outcome. Since in our dynamic game analyses we minimise costs (convergence), we have that a low value for the objective function (convergence function) indicates there is much welfare (convergence). As explained in section 5.3.3, we observe that the objective function values $J_i, i = 1, ..., 8$ are the same for the *NC* and the *CCO* case. Furthermore, we observe that in the *NB*-solution each Member State has substantially

Table 5.4: The objective function values, convergence value and weights in a fixed exchange rate regime

	Countries								
	Belgium	Denmark	Germ.	France	Ireland	Italy	Netherl.	U.K.	Conv.
NC	0.73	1.24	1.57	2.78	1.50	2.32	0.97	1.57	9.10
NCO	0.41	1.75	1.05	2.79	1.98	3.70	0.70	1.76	8.46
NB	0.36	0.91	0.63	1.60	1.28	1.40	0.74	1.28	6.71
CCO	0.73	1.24	1.57	2.78	1.50	2.32	1.97	1.57	3.24
	Weights								
	Belgium	Denmark	Germ.	France	Ireland	Italy	Netherl.	U.K.	Conv.
α^{NB}	0.13	0.15	0.05	0.04	0.22	0.05	0.21	0.16	-
α^{CCO}	0.16	0.03	0.06	0.08	0.02	0.19	0.31	0.14	0.42

more welfare than in both noncooperative solutions, *NC* and *NCO*. Also the *NB*-solution yields a substantially higher degree of convergence. This last aspect is remarkable since, in the *NCO*-outcome, each individual Member State additionally attributes some additional weight to minimise convergence, in contrary to the *NB*-solution. The experiments, where we tried different values for λ in the *NCO* game, did not change this result very much. In general, a higher λ, and hence a higher weight on convergence in the NCO-solution yielded more or less the same results with respect to the degree of convergence[10]. Furthermore, in comparison to the *NB*-solution, the *CCO*-solution yields an additional degree of convergence. These observations give already some evidence to the fact that coordination is a necessary requirement for convergence, or as stated by Brandsma and Italianer [9, page 11]:'...in the absence of proper coordination, a majority of the Member States would fail to meet the convergence criteria, no matter how seriously they tried to converge'.

The next interesting question is whether the convergence conditions diminish negative spillovers in the noncooperative case. If we compare the two noncooperative outcomes, we observe that welfare is higher in the *NCO* outcome for three countries (Belgium, Germany and the Netherlands) and lower for the other five countries (Denmark, France, Ireland, Italy and the U.K.). These results suggest that the convergence criteria in a noncooperative game increase total welfare for the three traditionally low inflationary countries and decrease total welfare for the five traditionally higher inflationary countries. If we again consider figure 5.4 and translate the problem to the eight dimensional case, then we observe that the convergence constraint moves the $NC' = NCO$ in the 'right' direction for the above mentioned three countries and in the 'wrong' direction for the other five countries.

If we recall the interpretation of the α^{NB}-weights in the *NB*-solution (see section 5.3.2), we see that Germany, France and Italy have a rather low weight. Hence, these countries gain most if a purely cooperative strategy is adopted. The relatively higher α^{NB}-weights for Belgium, Denmark, Ireland, the U.K. and the Netherlands may imply that these countries can put heavy pressure on EU-negotiations, since their welfare gains from playing cooperatively are less than those for the other three economies. These outcomes suggest that most gains come from cooperation between Germany and France. Internalising the spillover effects in a positive way seems therefore most profitable for these two interdependent countries. The dependent country Italy profits most from this cooperation between Germany and France, whereas all the other countries show only a slight increase in welfare. The explanation of the result that Germany and France profit most is that in a noncooperative world the additional costs Germany and France have to pay are for the greater part costs in terms of instruments and for a lesser part costs in terms of targets. The externalities are, however, only generated by the target variables and, therefore, the differences between the quantitative effects in the cooperative *NB*-solution or noncooperative solutions generated by the

[10]Changing λ from 0.2 to 0.6 yielded convergence values still higher than 8.0

larger countries are relatively small for the dependent countries. This observation explains also why small dependent countries like Belgium Denmark, Ireland and the Netherlands do not gain so much from a cooperative strategy [11]. The intuition behind this result for the third interdependent country, the U.K., is that, in the SLIM-model, this country is more dependent on the USA than on Germany and France. Traditionally the U.K. is more isolated than most other countries in the EU which is reflected by the rather weak interdependencies with the other countries in the SLIM-model.

We now turn to the *CCO*-outcome. The results with respect to the chosen convergence function seem quite good. First, this result suggests that if countries are willing to coordinate their policies, then a lot of convergence is possible. This observation follows from the low convergence value and the high weight on convergence in the *CCO* solution. This result suggests also that in figure 5.4, if we again translate the problem to the eight dimensional case, that the Pareto curve P' tends to move slowly to the right if we increase λ. It is possible to compare the 'welfare weights' α^{CCO} and α^{NB}. The rule of thumb one can apply here is: The Member States whose weights increase, from the *NB* to the *CCO* case, contribute more to the minimisation of the convergence function than the Member States whose weights decrease. If we apply this rule of thumb, then we see that Italy contributes very much to the minimisation of the convergence function. This result is not surprising since Italy is the country with the highest inflation rates and long term interest rates and, thus, the $C_i, i =$Italy, term contributes a lot to the convergence function C. Applying again the rule of thumb, we observe that Denmark, Ireland and the UK do not contribute very much to the minimisation of the convergence function. This result may imply that these three countries face more welfare loss when trying to converge since their problems are more structural, whereas the other five countries can already create a lot of convergence by 'simply' internalising their externalities.

In order to discuss more country specific results we present in table 5.5, for each country, the (average) target values and policy choices for the four different game outcomes over the planning period. The first observation is that optimal growth is found to be moderate and that, except for Ireland, average growth is comparable to the average growth values during the eighties. These results are obtained with, on average, lower levels of government expenditure and lower levels of the short (and long term) interest rates than during the period 1981-1990. The results are in accordance with the broad economic policy guidelines of the EU in which a reduction of short and long term interest rates is proposed in the short to medium term and budgetary consolidation should be achieved by reducing government

[11]This result depends, of course, strongly on the simplifying assumption of the interdependencies in the SLIM-model in which Belgium and the Netherlands are modelled as dependent economies. If we would, for instance, consider the total Benelux-economy, then the impact of this economy on France and Germany might be substantial.

Table 5.5: The average target- and instrumental values (1992-1999)[a]

		Bel.	Germ.	Denm.	France	U.K.	Irel.	Italy	Neth.
	Target values								
ΔY	*NC*	1.86	2.51	1.40	2.26	2.64	4.60	2.34	1.47
	NCO	1.94	2.56	1.50	2.61	2.70	4.69	2.40	1.67
	NB	1.85	2.46	1.53	2.12	2.58	4.55	2.43	1.48
	CCO	1.66	2.20	1.44	1.76	2.55	4.62	2.22	1.22
ΔPy	*NC*	3.52	4.15	5.26	5.67	4.92	5.97	8.36	3.80
	NCO	3.78	4.14	5.40	6.15	5.00	6.02	9.10	4.09
	NB	3.01	3.76	4.94	4.28	4.55	5.74	7.32	3.37
	CCO	2.92	3.70	4.70	3.92	4.37	5.62	6.08	3.30
ΔPc	*NC*	3.67	3.71	5.41	5.66	4.77	5.10	8.90	3.76
	NCO	3.98	3.71	5.54	6.12	4.86	5.15	9.72	4.05
	NB	2.90	3.40	5.06	4.30	4.40	4.82	7.71	3.21
	CCO	2.75	3.32	4.82	3.95	4.23	4.70	6.38	3.11
ΔW	*NC*	5.50	5.43	6.82	7.14	7.48	9.10	9.89	4.84
	NCO	5.91	5.46	7.06	7.82	7.60	9.18	10.74	5.32
	NB	4.68	4.91	6.45	5.50	7.02	8.85	8.77	4.27
	CCO	4.50	4.66	6.09	4.97	6.79	8.77	7.31	4.08
ΔN	*NC*	0.01	0.35	0.36	0.33	-0.34	0.68	0.76	0.01
	NCO	0.02	0.39	0.38	0.50	-0.30	0.73	0.79	0.07
	NB	0.03	0.52	0.44	0.24	-0.43	0.64	0.77	0.08
	CCO	-0.06	0.47	0.43	0.08	-0.52	0.64	0.69	-0.06
RL	*NC*	7.92	6.97	9.60	10.34	8.72	8.93	11.80	8.02
	NCO	8.36	7.10	8.00	9.22	8.42	8.03	10.23	7.45
	NB	8.49	8.16	9.42	10.34	9.52	8.95	11.49	8.18
	CCO	8.85	8.91	9.54	9.96	9.66	9.48	10.86	8.71
	Instrumental values								
		Bel.	Germ.	Denm.	France	U.K.	Irel.	Italy	Neth.
RS	*NC*	6.59	4.71	8.39	9.71	8.27	8.95	11.27	6.78
	NCO	7.33	4.91	6.44	8.03	7.66	7.51	9.42	5.89
	NB	7.47	7.08	8.40	9.95	10.00	8.96	10.89	6.89
	CCO	8.17	8.67	8.66	9.31	10.30	9.67	10.08	7.89
G	*NC*	15.099	14.149	13.135	15.124	12.410	9.389	13.620	12.655
	NCO	15.108	14.146	13.134	15.117	12.414	9.389	13.610	12.663
	NB	15.112	14.132	13.136	15.095	12.412	9.389	13.623	12.669
	CCO	15.121	14.129	13.124	15.078	12.410	9.389	13.594	12.674

[a] $\Delta Y, \Delta P_y, \Delta P_c, \Delta W, \Delta N$ in % growth per annum. *RL* and *RS* in % and *G* in logaritmic of the real level government expenditure values.

expenditure and by an improvement of the efficiency of the fiscal system (see, e.g., the 1994 broad economic guidelines [16]).

Studying some results more specifically we observe that for all countries the cooperative outcomes show lower inflation rates than the noncooperative outcomes. These outcomes are generated, on average, by reductions in government expenditure and an increase in the long term interest rates. As can be seen in table 5.5, both policies hamper growth in the SLIM-model. This property suggests that countries should use a higher interest rate policy in order to prevent inflationary growth [12].

For Germany we observe that in the cooperative setting it has to reduce domestic inflation in order to reduce foreign inflation. Because GDP-growth is strongly inflationary in the SLIM-model, this policy leads to lower GDP growth rates. Since Germany plays a leading role due to its large spillovers, we find in the cooperative game that Germany likes to prevent negative effects for other EU-economies, which is ultimately profitable for Germany itself. Important to note is the accumulation of inflation over time in the SLIM-model. A relatively high inflationary policy in Germany generates higher inflations abroad. In the next period this high inflation is transported back to Germany which, in the next period again, is transported back abroad and so on. The longer the planning period the more important this inflationary accumulation effect drives the final results. Remarkable is the policy change that occurs in Germany if the cooperative setting is replaced by a noncooperative one. In the cooperative solution Germany increases the short term interest rate and uses a contractionary fiscal policy in order to reduce inflation (and, thus, also inflation abroad) whereas in the noncooperative setting it does not care about the foreign effects and chooses a contractionary monetary and expansionary fiscal policy. This noncooperative behaviour leads to more growth in Germany but also to higher domestic and foreign inflation and lower domestic employment.

To understand most of the results we have to discuss the impact of the two policy instruments in the SLIM-model, the short term interest rate and government expenditure. We take Germany as an example. An increase in the German short term interest rate leads in general to an increase in the domestic long term interest rate. This results in a decline in domestic output, inflation and wages. There is, however, through the channel of the long term interest rate a stabilising effect, since the decline of inflation leads to lower interest rates. The initial increase in the long term interest rate in Germany has, furthermore, an increasing effect on the foreign long term interest rates and, hence, a negative effect on foreign growth. This effect can be valuable for these countries in order to fight against

[12]The same assesment was made by the monetary authorities in 1989: in a period of inflationary pressures, where the rate of growth exceded the level required to stabilize employment, nominal short-term interest rates increased in every EU-country (see Drèze en Malinvaud [23]).

inflation. On the other hand, a contractionary fiscal policy in Germany leads also to a decline in growth and inflation as well in Germany as in the foreign countries; but this decrease in inflation leads, through the interest rate channel, to a decline of the long term interest rate in Germany and also abroad. This last effect has again some increasing impact on growth and inflation. It is exactly this opposite functioning in policy behaviour between the short term interest rate and government expenditure which should be kept in mind when interpreting the results.

If we again compare the noncooperative and cooperative outcomes we see, on average, that the noncooperative outcomes yield higher growth rates and to a lesser extent higher employment rates but also to higher inflation and wage rates. This effect is mainly created by the four larger economies, in particular we see a substantial reduction in the short term interest rate in Germany and the UK and a substantial expansionary fiscal policy in Germany and France.

Another interesting aspect is that the *CCO*-outcome suggests that countries with traditionally low interest rates, such as Germany, Belgium and the Netherlands should adjust their interest rates to higher levels in order to achieve global convergence. Our model suggests that it is less costly for the EU as a whole to reach convergence if these three countries adjust their interest rate levels upwards, since this would make it easier for the other five countries, which have traditionally higher interest rates, to achieve these lowest three rates. The noncooperative outcome, however, suggests that Germany, the Netherlands, and to a lesser extent Belgium, should follow more the policy they advocated during the eighties where the interest rates were, for at least Germany and the Netherlands, substantially lower than for the other five countries. This raises the important question: 'who should converge to who?'. It is clear that in a Treaty with *fixed* policy targets this question would not exist but in this case, where we have *flexible* policy targets, this is a serious issue. One could argue that this is one of the reasons why Germany is such an advocate of a more speed EU, since, in that case, they could follow, more or less, their noncooperative strategy with low interest rates and, hence, leaving the other countries the option to follow or not. It is clear that such a noncooperative strategy of Germany saddles up most other countries with more costs than if a cooperative strategy was adopted by Germany. These increasing costs in the foreign countries are, partly, transferred back to Germany which in the end yields higher costs for Germany as well in the noncooperative case. This argument may also explain why Germany uses this threat-argument of a two speed Europe in its negotiations with the other EU-economies. On the other hand, the low gains the model predicts for the UK, if they play cooperatively, suggests that the UK can put heavy pressure on the negotiations since they can not gain much during Stage Two. This may be an explanation why it threatens, once in a while, with leaving the EU. Remark, that we disregard the possible profits for each country which it expects to gain in Stage Three of EMU [13]. Before proceeding with our analyses we first give the dynamic game results in the tight exchange rate regime.

5.5.2 Empirical results in a tight exchange rate regime

We present for this regime the same tables as shown in the previous subsection. In table 5.6 we present the implications for welfare for the four game outcomes. The figures can be

Table 5.6: The objective function values, convergence values and weights in a tight exchange rate regime

	Countries								
	Belgium	Denmark	Germ.	France	Ireland	Italy	Netherl.	U.K.	Conv.
NC	1.05	1.01	2.96	1.93	1.12	2.34	1.08	1.27	8.76
NCO	0.88	1.15	1.99	1.63	1.18	2.82	0.94	1.27	8.32
NB	0.34	0.73	1.19	1.21	0.92	1.48	0.72	1.01	6.63
CCO	1.05	1.01	2.96	1.93	1.12	2.34	1.08	1.27	2.53
	Weights								
	Belgium	Denmark	Germ.	France	Ireland	Italy	Netherl.	U.K.	Conv.
α^{NB}	0.07	0.18	0.03	0.07	0.25	0.06	0.14	0.20	-
α^{CCO}	0.10	0.04	0.06	0.12	0.06	0.19	0.26	0.17	0.44

compared to the outcomes presented in table 5.4. If in the tight exchange rate regime it would be optimal that no country uses the possibility of managing its exchange rate with the dollar, i.e. $\Delta E = 0$ for the whole planning period, then we would obtain exactly the same outcomes as in the fixed exchange rate regime which we considered in the previous subsection.

If we compare the two noncooperative outcomes we see that in a tight exchange rate regime the *NCO* outcome is profitable for four countries (Belgium, Germany, France, the Netherlands), malicious for three countries (Denmark, Ireland, Italy) and makes no difference for the UK. Furthermore, these results suggest a small increase in global welfare since the total gains of the four countries seem to be higher than the total losses of the three countries.

[13] If the Member States would consider the possible positive gains of Stage Three, then it is likely that each Member State is willing to accept more costs during Stage Two. In our context, this means that Member States are willing to consider outcomes outside the negotiation area where even more convergence would be possible.

Concerning individual welfare in the *NCO*-solution we noticed that increasing λ yielded substantial gains for the four mentioned countries and the UK, whereas for the other three countries we found that the objective function values remained almost the same as shown in table 5.6, with $\lambda = 0.2$. The implications for total welfare depend, of course, on the weights one assigns to the individual welfare functions, but with equal weights we found that total welfare substantially increased when increasing λ. These experiments suggest that the outcomes severely depend on the specification of the convergence conditions in a noncooperative world and that it even may be possible that a particular convergence function could be constructed in which each individual Member State would be better off than in the NC-solution. Comparing tables 5.5 and 5.7, we observe that managing the exchange rate in the SLIM-model is profitable for the five traditionally higher inflationary countries but malicious for Belgium, Germany and the Netherlands. This observation holds for any of the four game outcomes.

Remarkable are the differences in welfare outcomes in the noncooperative case for Germany and France between the tight and the fixed exchange rate regime. A great deal of burden, associated with the various exchange rate policies in the tight exchange rate regime, is covered by Germany. This observation follows from the fact that Germany has a substantially lower welfare value in the fixed exchange rate regime than in the tight exchange rate regime, whereas for France it is the other way around. The reason for this finding is that in the SLIM-model, French output is strongly positively affected by an appreciation of the nominal exchange rate between the French Franc against the Dollar and a depreciation of the nominal exchange rate between the German mark against the Dollar.

The overall results of table 5.6 are in line with those of table 5.4. We observe also that the weights α^{NB} and α^{NC} are (very) similar in both dynamic games experiments. Again Germany, France and Italy have the lowest weight values in the *NB*-solution. The four smaller dependent countries and the more isolated country UK gain less in a cooperative strategy. In the cooperative solutions, the fixed exchange rate regime can be seen as a restricted form of the tight exchange rate regime and should therefore by definition lead to lower total welfare values. If we, as an example, multiply all individual weights with the corresponding individual welfares then we find that total welfare increases from 0.89 in the tight exchange rate regime to 0.98 in the fixed exchange rate regime. If we use the same α^{NB}-weights then we find that in the NC case (and, thus, also in the CCO case) that total welfare is 1.30 in the tight exchange rate regime and 1.36 in the fixed exchange rate regime. For the *CCO*-solution this observation implies that in the tight exchange rate regime 1.30-0.89=0.41 of total welfare is used for minimizing convergence and in the fixed exchange rate regime 1.36-0.98=0.38 of total welfare. It is clear that the *CCO*-solution chosen in this example does not guarantee that the convergence criteria are reached. It is, however, possible to search for strategies which satisfy the convergence criteria and a 'rough measure' of convergence costs, in terms of welfare, could then be constructed as shown by the example above.

In table 5.7 we present the averages of the target and instrumental values for each country seperately. A first glance shows that the qualitative outcomes of the fixed exchange rate regime are similar to the tight exchange rate regime. For all countries we see again that inflation is reduced in the cooperative case. We observe the strongest adjustments in the exchange rates in the *CCO* outcome. For this outcome, we observe that the traditionally higher inflationary countries, such as France, Ireland and Italy, appreciate their currency against the dollar and that the traditionally low inflationary countries, such as Belgium, Germany and the Netherlands depreciate their currency, in order to fulfil the convergence requirements (of especially convergence in consumer price inflation).

Now let us take a closer look at the four convergence criteria. Since we have only one fiscal policy instrument, government expenditure, the experiments are less useful for checking the budget criteria. We did, however, construct the desired government expenditure paths such that all Member States substantially reduced government expenditure in order to be able to restore their budget. In comparison with the other game outcomes, for most countries the *CCO*-outcome yielded lower levels of government expenditure. This was not the case for Belgium and the Netherlands. Comparing the noncooperative and the cooperative outcomes we see that both countries changed their policy behaviour in both experiments. In the noncooperative game they follow a contractionary fiscal policy and a low interest rate policy. However, in especially the *CCO*-case, they have to follow the higher interest rates of the other countries and, therefore, both countries have to increase interest rates. This policy has, however, a negative impact on output growth and in order to offset some of this negative impact both countries react with an increase in government expenditure.
Through the high priority of 10 on the exchange rates we obviously find that the exchange rates are kept very tight around the 1991 values. In table 5.7 one observes that most values of ΔE are close to zero; we only find a substantial depreciation of the German mark against the Dollar in the case of the *CCO*-outcome. It is clear that such a policy is very unlikely in reality. However, in the cooperative *CCO*-outcome studied here, Germany is very much concerned about convergence of *all* the EU-Member States and, therefore, reacts with a depreciation policy.
In order to check the two central criteria of convergence in consumer price inflation and nominal long term interest rates we will present some graphs. In figure 5.5, we present the consumer price deflator responses for three different game outcomes in the tight exchange

Table 5.7: The average target- and instrumental values (1992-1999)[a]

		Bel.	Germ.	Denm.	France	U.K.	Irel.	Italy	Neth.
		Target values							
ΔY	NC	1.86	2.52	1.41	2.35	2.65	4.64	2.46	1.49
	NCO	1.97	2.58	1.48	2.59	2.68	4.69	2.51	1.50
	NB	1.87	2.47	1.57	2.20	2.56	4.59	2.48	1.63
	CCO	1.74	2.20	1.43	1.90	2.52	4.49	2.23	1.17
ΔPy	NC	3.33	4.11	5.25	5.56	4.92	5.91	8.25	3.64
	NCO	3.54	4.09	5.32	5.76	4.94	5.92	8.61	3.80
	NB	3.08	3.68	4.92	4.52	4.63	5.71	7.48	3.39
	CCO	3.09	3.75	4.76	4.11	4.45	5.54	5.96	3.45
ΔPc	NC	3.48	3.66	5.39	5.54	4.78	5.03	8.75	3.60
	NCO	3.69	3.65	5.45	5.74	4.81	5.04	9.15	3.75
	NB	2.99	3.36	5.04	4.52	4.48	4.77	7.89	3.22
	CCO	2.96	3.39	4.88	4.12	4.31	4.57	6.25	3.28
ΔW	NC	5.23	5.37	6.82	7.06	7.50	9.06	9.85	4.65
	NCO	5.54	5.41	6.96	7.39	7.54	9.08	10.26	4.92
	NB	4.79	4.82	6.44	5.81	7.11	8.82	8.98	4.28
	CCO	4.76	4.72	6.18	5.22	6.86	8.61	7.23	4.25
ΔN	NC	0.03	0.39	0.36	0.38	-0.32	0.70	0.81	0.09
	NCO	0.07	0.43	0.37	0.50	-0.29	0.72	0.83	0.16
	NB	0.04	0.56	0.45	0.30	-0.41	0.66	0.79	0.10
	CCO	-0.03	0.46	0.42	0.15	-0.49	0.60	0.70	-0.07
RL	NC	8.07	7.07	9.56	10.05	8.72	8.69	11.30	8.03
	NCO	8.11	7.07	8.66	9.28	8.59	8.25	10.49	7.74
	NB	8.46	8.34	9.39	10.09	9.42	8.71	11.09	8.17
	CCO	8.93	8.91	9.50	9.91	9.60	9.33	10.69	8.78
		Instrumental values							
		Bel.	Germ.	Denm.	France	U.K.	Irel.	Italy	Neth.
RS	NC	6.84	4.92	8.36	9.36	8.29	8.48	10.62	6.82
	NCO	6.96	4.89	7.27	8.25	8.09	7.78	9.68	6.40
	NB	7.44	7.43	8.38	9.59	9.77	8.49	10.36	6.89
	CCO	8.28	8.67	8.58	9.23	10.13	9.51	9.88	8.00
G	NC	15.091	14.147	13.136	15.119	12.410	9.389	13.620	12.655
	NCO	15.095	14.144	13.135	15.110	12.412	9.389	13.615	12.658
	NB	15.112	14.125	13.136	15.099	12.414	9.389	13.623	12.669
	CCO	15.126	14.128	13.128	15.086	12.411	9.388	13.598	12.675
ΔE	NC	-0.02	0.00	0.00	-0.02	0.00	-0.06	-0.07	0.00
	NCO	0.00	0.00	0.00	-0.03	0.00	-0.06	-0.09	0.00
	NB	-0.02	0.13	0.00	-0.01	0.06	-0.05	-0.06	0.00
	CCO	0.15	0.54	0.00	-0.09	0.01	-0.14	-0.20	0.02

[a] $\Delta Y, \Delta P_y, \Delta P_c, \Delta W, \Delta N, \Delta E$ in % growth per annum. RL and RS in % and G in logaritmic of the real level government expenditure values.

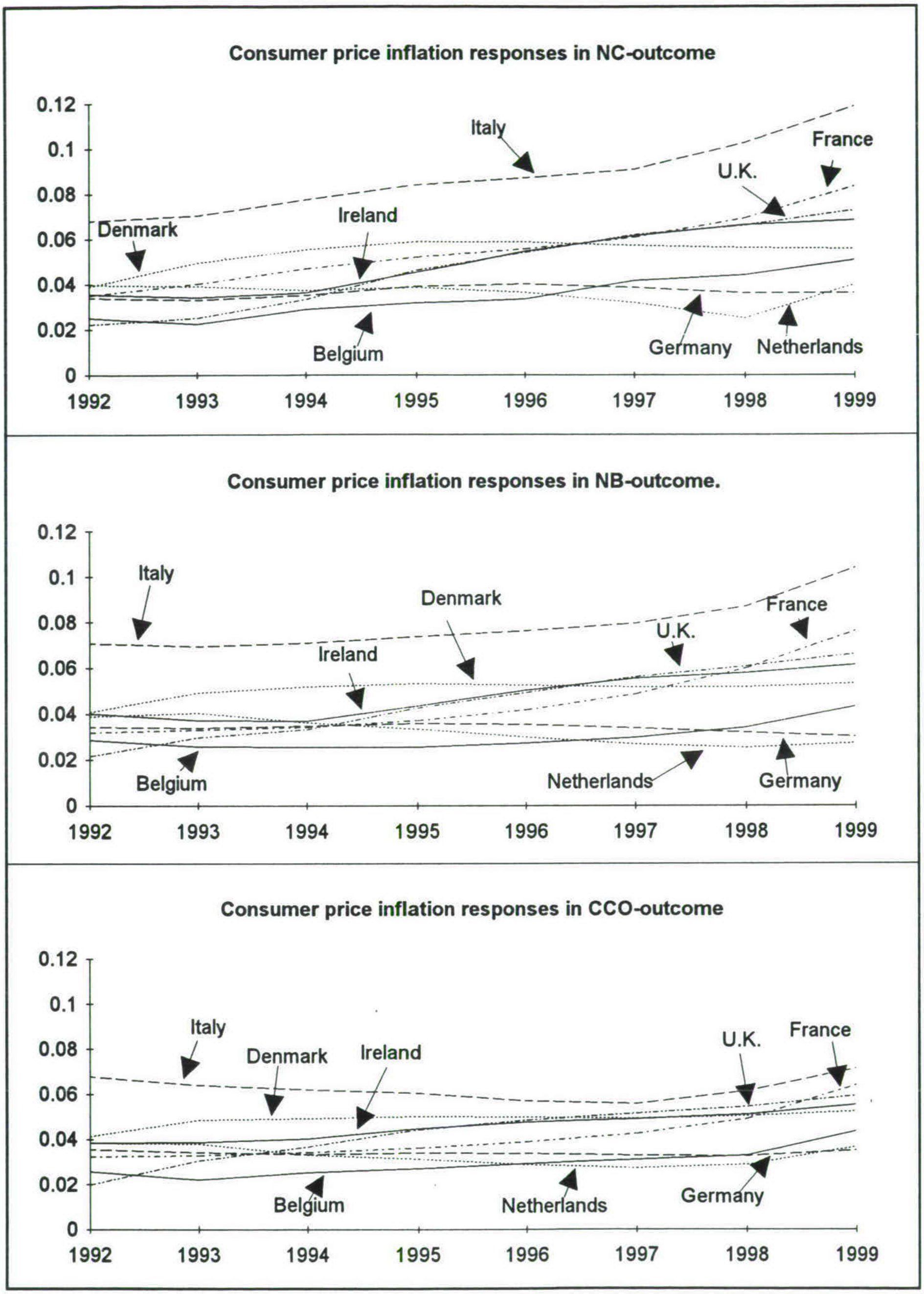


Figure 5.5: Consumer price inflation responses in various game outcomes

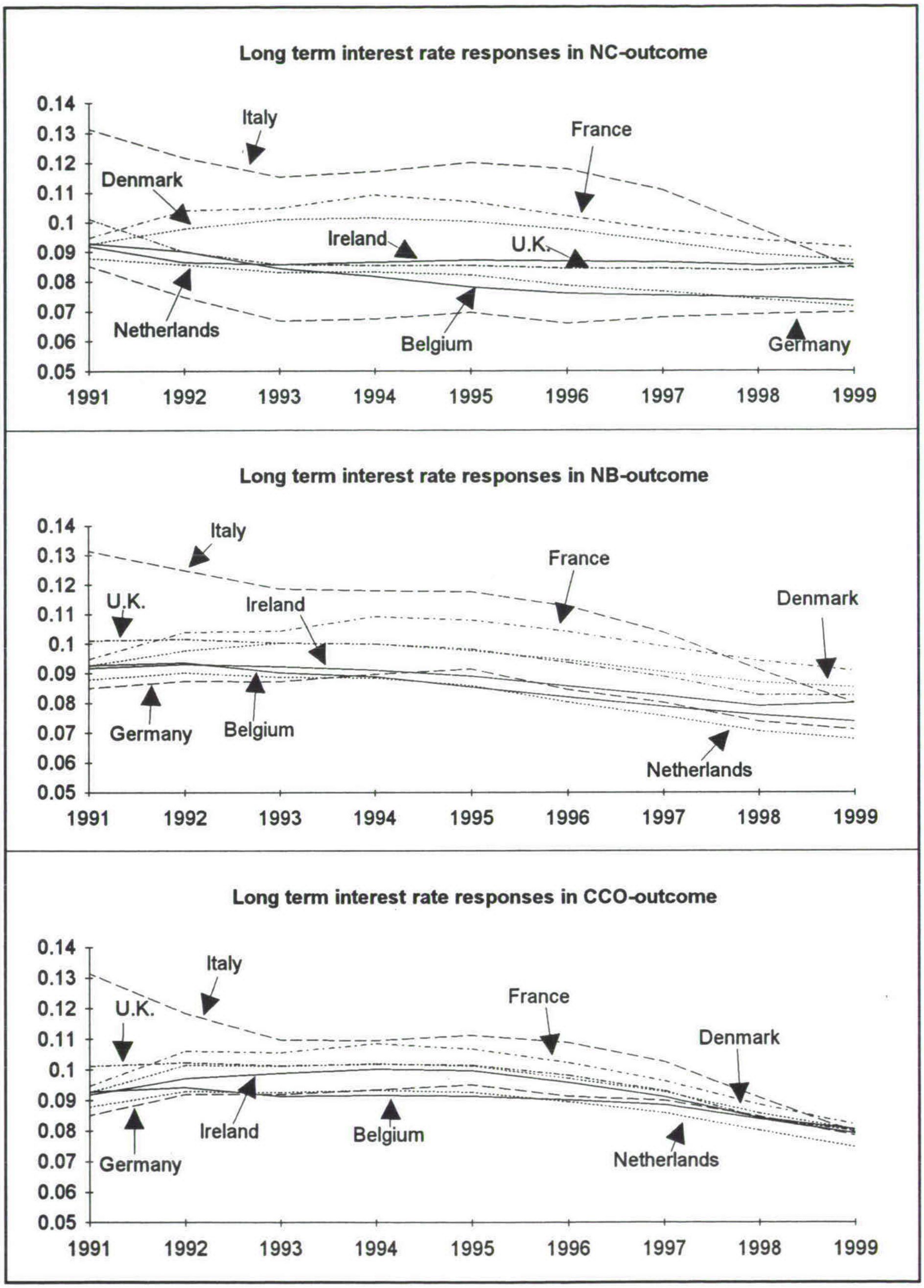


Figure 5.6: Long term interest rate responses in various game outcomes

rate regime[14]. In figure 5.6 we present the long term interest rates for the same game outcomes. The graphs show some interesting facts. First of all, the graphs suggest that it seems to be much harder to converge for consumer price inflation than for the long term interest rates. This fact is, however, more a property of the model and is related to the fact that long term interest rates in the SLIM-model can directly be manipulated by the short term interest rates, whereas the consumer price deflator can only indirectly be influenced [15]. As already indicated by the convergence values of table 5.6, we see in all figures that the degree of convergence increases from the *NC*, *NB* to the *CCO*-outcome. The long term interest rate criterion is fulfilled by all the Member States in the *CCO*-outcome with an average long term interest of 8.5%. For the consumer price deflator we find less convergence. If we consider the three Member States with the lowest inflation (Belgium, Germany and the Netherlands), then we find a consumer price deflator of around 4%. Since, the Maastricht Treaty allows only for consumer price inflation rates which are no more than 1.5% points above the average for the three countries with the lowest inflation rates we see that three of the other five Member States fulfil this rule and that Italy and France come very close to the 5.5% norm. The results of the graphs are in accordance with the empirical results of Brandsma and Italianer [9] who state that, with appropriate coordination, all the eight Member State should be able to fulfil the convergence criteria. In that study the authors used the European Commission's Quest model, which contains all the EU-Member States and they allowed sometimes, in order to fulfil the criteria, for drastic measures such as dismissing government employees or using the wage rate of the government employees as a policy variable. Remark, however, that in that research the broad economic policy guidelines [16] were followed which propose inflation rates of 2-3%. It is important to stress that these inflation rates are low if one compares them with the average inflation rates during the eighties. Therefore, it is important to realise that during the 1990-1994 period average output was rather low in the EU, so that several countries fighted successfully against inflation. A property of the SLIM-model, and also of most other EU-models (see, e.g. Douven and Plasmans [19]), is that growth is strongly inflationary (in the long run on average 1% GDP-growth yields about 2% inflation). This relationship between growth and inflation in most models may be somewhat exaggerated, but also may suggest that countries will get a hard time if overall growth in the EU substantially increases. Concerning employment, tables 5.5 and 5.7 suggest also that in order to reach the convergence criteria employment will decline in most countries, except for Denmark and Germany. For these two countries the decline in wages, which stimulates employment, was large enough to offset the decrease

[14] The similarity between the graphs of the *NC* and *NCO* outcome was large, so we decided to include the graph of the *NC* outcome only

[15] For instance, if we would follow the same strategy as used in Brandsa and Italianer [9], who consider wages as an instrumental variable, then it would be much easier to obtain convergence in (lower) consumer price inflation rates.

in output, which hampers employment. In all experiments the impact on employment is rather low which suggests that the policy measures used in the SLIM-model are not adequate enough to fight for substantial increases in employment. It suggests that more structural changes are needed in order to promote employment in several EU-economies.

5.6 Conclusions

In this chapter we carried out a dynamic game analysis with the SLIM-model. In the dynamic game we compare four (hypothetical) scenarios. First, a noncooperative scenario which is represented by the feedback Nash solution (*NC*) in which each country minimises its own welfare and, second, a feedback Nash solution (*NCO*) in which each country minimises its own welfare, but additionally tries to fulfil the two convergence criteria of convergence in the long term interest rates and convergence in consumer price inflation rates. Third, a purely cooperative scenario, which is represented by the Nash bargaining solution (NB) and, fourth, a cooperative convergence scenario (CCO). In this last scenario the EU-Member States play in a cooperative mode, but face a dynamic constraint of convergence in consumer price inflation and long term interest rates. These two convergence conditions are elaborated at the Maastricht Treaty (1991). The third condition in this Treaty is that each Member State should strive for a sustainable government financial position. This aspect is modelled by means of the individual welfare functions of each country. In our experiments we assumed that each country substantially lowers its government expenditure, in order to restore its government deficit. The fourth Maastricht convergence condition, no exchange rate realignments for at least two years, is modelled by keeping in one experiment the (dollar) exchange rate fixed at the initial 1991 levels and in a second experiment by allowing only for very small movements around these 1991 levels. For the *CCO*-solution we assume that countries do not accept 'welfare losses' which are higher than the 'welfare costs' obtained in the noncooperative solution. This assumption makes it possible to prove that the maximum convergence that can be reached is limited. Furthermore, one can obtain a unique cooperative convergence solution in this case.

The first important observation is that we found some evidence that convergence does not occur if the EU-Member States do not coordinate their policies (see also Brandsma and Italianer [9]).
Furthermore, our theoretical study suggests that one should design (optimal) convergence criteria in the sense that the impact of the convergence criteria is profitable in the noncooperative case and remains close to Pareto optimal solutions in the cooperative case. Since, in reality, we observe a mix of cooperative and noncooperative policy behaviour this study

suggests that the European Commission should strive for restrictions on national policies in which negative spillovers diminish in the noncooperative game but still keep almost all the gains in the cooperative game. Our first empirical results give some evidence to the fact that the Maastricht criteria at least do not harm much in a noncooperative setting and are indeed close to Pareto optimal solutions in the cooperative setting.
The model 'predicts' a nominal long term interest rate of around 8.5% and a consumer price deflator of around 4% as optimal in 1999. Optimal growth will in all countries be moderate.
Country specific remarks are that the SLIM-model predicts that the two largest EU-Member States, Germany and France gain most when comparing the noncooperative outcome with the purely cooperative outcome. This gives some evidence to the fact that strong (more independent) countries gain more, by playing cooperatively, than small (more dependent) countries. The main intuition behind this result is that the gains of the strongly interdependent countries are mainly due to a more effective use of their instrumental variables in order to produce, more or less, the same target variables, whereas the generated spillovers to the dependent countries are only produced by the target variables.
The two convergence conditions of the long term interest rate and the consumer price inflation are examples of flexible shared targets, since it is beforehand not clear what the ideal target values in 1999 will be. This raises an important question: Who converges to who? Should countries strive for the low targets advocated by Germany or should Germany adjust its interest rate and inflation targets to higher values and, thus, giving the other countries more room (and, hence, less welfare loss) for achieving the criteria. Our model predicts that it would be less costly for the EU as a whole if the traditionally low interest rate countries Belgium, Germany and the Netherlands converge towards the higher interest rates of the five other EU-economies, instead of using as convergence target a fixed low nominal interest rate level. Inflation targets can, more or less, be fixed on the initial 1991 values of around 2-4%. We have to emphasize that this result hinges decisively on the assumption that all countries (including Italy) should converge. Excluding, e.g., an inflationary country like Italy from a cooperative dynamic game experiment would lead to lower inflation target rates for the other players in that game.

It is important to stress that the results obtained in this chapter are, of course, model dependent. Further research, such as robustness and sensitivity analyses, is desirable in order to obtain a better understanding of the different game outcomes. In particular the following aspects should be elaborated:
(1) In our research we assumed exogenous behaviour of the two foreign countries, USA and Japan. How will the dynamic game outcomes change if we endogenise their behaviour?
(2) Since it is extremely difficult to model the exact specifications of the convergence conditions of the Maastricht Treaty we modelled convergence in this chapter by assuming that

the EU-Member States converge to the average long term interest rate level and average consumer price inflation of the larger three EU-Member States, Germany, France and U.K. This aspect of the model should be elaborated more and it may be interesting to look for 'optimal' convergence functions.

Appendix A

Appendices of chapter 3

A.1 Proof of Theorem 3.2.1

Consider N strictly concave and twice differentiable functions $J_i(u), i = 1, ..., N$, for $u \in U$. Now the Pareto curve is determined by a set $\alpha_i > 0, i = 1, ..., N$, and the following problem:

$$\max_{u \in U} \alpha_1 J_1(u) + \cdots + \alpha_1 J_1(u). \tag{A.1}$$

Without loss of generality we assume in the sequel that $\alpha_i > 0, i = 1, ..., N$ and $\sum_{i=1}^{N} \alpha_i = 1$. Now every element, say $(\alpha_1^*, ..., \alpha_N^*)$, determines a unique strategy u^* and a unique point $J^* = J(u^*)$ on the Pareto curve. Thus

$$\underline{\alpha} := \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_{N-1} \end{pmatrix} \rightarrow \begin{pmatrix} J_1^*(\underline{\alpha}) \\ \vdots \\ J_N^*(\underline{\alpha}) \end{pmatrix}$$

Thus we can write:

$$\left\{ \begin{array}{c} x_1 = J_1^*(\underline{\alpha}) \\ \vdots \\ x_N = J_N^*(\underline{\alpha}) \end{array} \right. , \text{or} \qquad \left\{ \begin{array}{c} x_1 - J_1^*(\underline{\alpha}) = 0 \\ \vdots \\ x_N - J_N^*(\underline{\alpha}) = 0 \end{array} \right.$$

The next step is to write x_N as a function of $x_1, ..., x_{N-1}$. Therefore, we use the previous first $N-1$ equations and write implicitly $\underline{\alpha} = \underline{\varphi}(x_1, ..., x_{N-1})$. Using, now the implicit function theorem, we have that: $\underline{\varphi}' = -\mathcal{J}_{\underline{\alpha}}^{-1}$ where

$$\mathcal{J}_{\underline{\alpha}} = \begin{pmatrix} -\frac{\partial J_1}{\partial \alpha_1} & \cdots & -\frac{\partial J_1}{\partial \alpha_{N-1}} \\ \vdots & & \vdots \\ -\frac{\partial J_{N-1}}{\partial \alpha_1} & \cdots & -\frac{\partial J_{N-1}}{\partial \alpha_{N-1}} \end{pmatrix}.$$

Thus,

$$\begin{aligned} x_N &= J_N^*(\underline{\varphi}(x_1, ..., x_{N-1})) \\ &= J_N^*(\varphi_1(x_1, ..., x_{N-1}), ..., \varphi_{N-1}(x_1, ..., x_{N-1})). \end{aligned}$$

Using the chain rule we have that:

$$\frac{\partial x_N}{\partial x_i} = \frac{\partial J_N}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial x_i} + \cdots + \frac{\partial J_N}{\partial \varphi_{N-1}}\frac{\partial \varphi_{N-1}}{\partial x_i}$$

Now, since u^* is a solution of (A.1) we have that (envelop theorem):

$$\alpha_1 \frac{\partial J_1}{\partial \alpha_i} + \cdots + \alpha_N \frac{\partial J_N}{\partial \alpha_i} = 0,$$

for $i = 1, ..., N-1$. Therefore,

$$\begin{aligned} \frac{\partial x_N}{\partial x_i} &= -\frac{1}{\alpha_N}(\sum_{i=1}^{N-1} \alpha_i \frac{\partial J_i}{\partial \alpha_1})\frac{\partial \varphi_1}{\partial x_i} - \cdots - \frac{1}{\alpha_N}(\sum_{i=1}^{N-1} \alpha_i \frac{\partial J_i}{\partial \alpha_{N-1}})\frac{\partial \varphi_{N-1}}{\partial x_i} \\ &= -\frac{1}{\alpha_N}(\alpha_1, ..., \alpha_{N-1}) \begin{pmatrix} -\frac{\partial J_1}{\partial \alpha_1} & \cdots & -\frac{\partial J_1}{\partial \alpha_{N-1}} \\ \vdots & & \vdots \\ -\frac{\partial J_1}{\partial \alpha_1} & \cdots & -\frac{\partial J_{N-1}}{\partial \alpha_{N-1}} \end{pmatrix} \begin{pmatrix} -\frac{\partial \varphi_1}{\partial x_i} \\ \vdots \\ -\frac{\partial \varphi_{N-1}}{\partial x_i} \end{pmatrix} \\ &= -\frac{1}{\alpha_N}(\alpha_1, ..., \alpha_{N-1}) \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \leftarrow i^{th}\text{place} \\ &= -\frac{\alpha_i}{\alpha_N} \end{aligned}$$

which yields the proof.

A.2 Proof of Theorem 3.3.2

Since we assumed that $J_i, i = 1, ..., N$ are strictly concave and twice differentiable and that U^P lies in the interior of U we have that for each α a strategy is uniquely determined by the following first order conditions:

$$\begin{aligned} \alpha_1 J_{11} + \alpha_2 J_{21} + \cdots + \alpha_N J_{N1} &= 0, \\ \alpha_1 J_{12} + \alpha_2 J_{22} + \cdots + \alpha_N J_{N2} &= 0, \\ &\vdots \\ \alpha_1 J_{1N} + \alpha_2 J_{2N} + \cdots + \alpha_N J_{NN} &= 0, \end{aligned} \tag{A.2}$$

where $J_{ij} = \partial J_i/\partial u_i$. Solving these equations yields $u^*(\alpha)$. Using the simplifying notation:

$$\tilde{J}_{ij} = J_{j1}u_{1i} + J_{j2}u_{2i} + \cdots + J_{jN}u_{Ni}$$

for $i, j \in 1, ...N$, with $u_{ij} = \partial u_i/\partial \alpha_j$ and rearranging (A.2) we see that:

$$\alpha_1\tilde{J}_{1i} + \alpha_2\tilde{J}_{2i} + \cdots + \alpha_N\tilde{J}_{Ni} = 0 \tag{A.3}$$

for $i \in 1, ...N$. This is an important relationship which holds for all Pareto optimal solutions. Next we consider the first order conditions from maximizing the Nash-product (3.4). They are (with the simplifying notation $s_i = J_i - d_i$):

$$\tilde{J}_{1i}s_2s_3\cdots s_N + \tilde{J}_{2i}s_1s_3\cdots s_N + \cdots + \tilde{J}_{Ni}s_1s_2\cdots s_{N-1} = 0 \tag{A.4}$$

for $i = 1, ..., N$. Now comparing the two systems of N equations (A.3) and (A.4) we see that:

$$\begin{aligned} c\alpha_1 &= s_2s_3\cdots s_N \\ c\alpha_2 &= s_1s_3\cdots s_N \\ &\vdots \\ c\alpha_N &= s_1s_2\cdots s_{N-1}, \end{aligned}$$

where c is some constant, satisfies both systems of equations. Taking now into consideration that $\sum_{i=1}^N \alpha_i = 1$ we see that

$$\alpha_i = \frac{\prod_{i\neq j} s_i}{\sum_{i=1}^N \prod_{i\neq j} s_i} \tag{A.5}$$

satisfies both systems of equations. The Pareto strategy wich belongs to this α maximizes (3.2) and maximizes (3.4). Since the Nash bargaining solution determines a unique outcome $J \in P$ (see Nash [58]) and the fact that every strategy $u \in U^P$ is uniquely determined by an $\alpha \in [0, 1]$ we have that α is uniquely determined by this relationship. □

A.3 Proof of Theorem 3.4.1

In the first subsection we will first illustrate the proof for the 3-player case. The same arguments we use in the 3-player case will be used in the next subsection for the N-player case. An advantage of presenting the proof in this way is that the reader gets a better understanding of the proof and in particular of the truncated cube $C\backslash\{\cup_{i=1}^N A_i\}$.

A.3.1 The 3-player case

Without loss of generality, we take for the disagreement point d, the origin. Thus, assume d =(0,0,0). Then cube C is determined by the convex polytoop with 2^3 angular points

$$\{(x_1, x_2, x_3) \mid x_i \in \{0, J_i^I\}, \quad i = 1, 2, 3\},$$

and the three convex polytopes A_i, are described by the set of angular points $a_i, i = 1, 2, 3$:

$$\begin{aligned}
a_1 &= \{(0, 0, \tfrac{2}{3}J_3^I), (0, \tfrac{2}{3}J_2^I, 0), (0, 0, 0), (J_1^I, 0, \tfrac{2}{3}J_3^I), (J_1^I, \tfrac{2}{3}J_2^I, 0), (J_1^I, 0, 0)\}, \\
a_2 &= \{(0, 0, \tfrac{2}{3}J_3^I), (\tfrac{2}{3}J_1^I, 0, 0), (0, 0, 0), (0, J_2^I, \tfrac{2}{3}J_3^I), (\tfrac{2}{3}J_1^I, J_2^I, 0), (0, J_2^I, 0)\}, \\
a_3 &= \{(\tfrac{2}{3}J_1^I, 0, 0), (0, \tfrac{2}{3}J_2^I, 0), (0, 0, 0), (\tfrac{2}{3}J_1^I, 0, J_3^I), (0, \tfrac{2}{3}J_2^I, J_3^I), (0, 0, J_3^I)\}.
\end{aligned}$$

First, consider the Kalai-Smorodinsky solution. Remark that each convex polytope A_i contains $\frac{1}{3}J^I$ as an edge point. Furthermore, observe that the line through the origin and the ideal point λJ^I lies in the interior of each convex polytope A_i for $0 < \lambda < \frac{1}{3}$ and outside each A_i for $\frac{1}{3} < \lambda \leq 1$. Next, consider the convex polytoop D, determined by the angular points:

$$\{(J_1^I, 0, 0), (0, J_2^I, 0), (0, 0, J_3^I)\}$$

Now, it is easy to show, that the intersection of the line λJ^I and D occurs for $\lambda = \frac{1}{3}$. Since, the set of Pareto optimal solutions is concave and the fact that the edges of P lie in D, we have that the KS-solution is given by λJ^I, for some $\frac{1}{3} < \lambda \leq 1$. Combining the two results, we have in particular that the KS-solution lies inside cube C, but outside $A = \cup_{i=1}^{3} A_i$.

Secondly, consider the Nash bargaining solution. This solution is determined by maximizing the Nash-product $J_1 J_2 J_3$, with $J \in S$. Since, P is strictly concave we can write each $J_i, i = 1, 2, 3$, as a function of the other two components. First we consider the case where J_3 is written as a function of J_1, J_2, thus $J_3 = \varphi(J_1, J_2)$. Now, consider the function:

$$f(J_1, J_2) = J_1 J_2 \varphi(J_1, J_2)$$

with $J_i \in [0, J_i^I]$ for $i = 1, 2$. Note that the domain of φ is a convex set which can be parametrized using spherical coordinates. See figure A.1. That is, every $(J_1, J_2) \in H$ can be written as

$$J_1 = r\omega_1, \qquad J_2 = r\omega_2,$$

where $\omega = (\omega_1, \omega_2)$ is an element of the unit sphere $\Omega = \{(J_1, J_2) \mid J_1^2 + J_2^2 = 1\}$. Using this transformation, f reduces to

$$f(r, \omega) = r^2 \omega_1 \omega_2 \varphi(r, \omega)$$

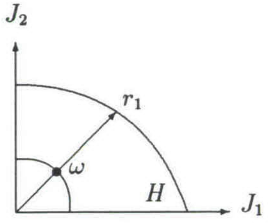


Figure A.1: The domain of φ in the 3-player case.

Now, for a fixed $\omega \in \Omega$ we look for the $r \in H$ that maximizes the Nash-product $J_1 J_2 J_3$. Assume that for this fixed ω the maximal possible r is r_1. So, assume $r \in [0, r_1]$. Then we can derive f'_r:

$$f'_r(r,\omega) = \{2r\varphi(r,\omega) + r^2\varphi'_r(r,\omega)\}\omega_1\omega_2.$$

Now, we are interested in points where $f'_r(r,\omega) = 0$ for $r > 0$. Since, ω is fixed the problem is equivalent with:

$$g(r) := 2\varphi(r,\omega) + r\varphi'_r(r,\omega) = 0.$$

Now, first observe that since φ is strictly concave we have that

$$g'_r(r) = 3\varphi'_r(r,\omega) + r\varphi''_{rr}(r,\omega) < 0.$$

Now, since g_r is monotone descending with $g(0) > 0$ and $g(r_1) < 0$ we have that $g(r)$ obtains a unique maximum between $[0, r_1]$. Using now the mean value theorem we have that for a $\xi \in [\frac{2}{3}r_1, r_1]$:

$$\begin{aligned} g(\frac{2}{3}r_1) &= 2\varphi(\frac{2}{3}r_1,\omega) + \frac{2}{3}r_1\varphi'_r(\frac{2}{3}r_1,\omega) \\ &= 2\varphi(\frac{2}{3}r_1,\omega) - 2\varphi(r_1,\omega) + \frac{2}{3}r_1\varphi'_r(\frac{2}{3}r_1,\omega) \\ &= -\frac{2}{3}r_1\{\varphi'_r(\xi,\omega) - \varphi'_r(\frac{2}{3}r_1,\omega)\} > 0. \end{aligned}$$

This implies that $g(r)$ has a zero in the interval $[\frac{2}{3}r_1, r_1]$.
Now, observe that this result can be obtained for every $\omega \in \Omega$. Since P is strictly concave we have that (r,ω) covers at least the area of the convex surface determined by the angular points $\{(0,0), (J_1^I, 0), (0, J_2^I)\}$. Thus we have that the maximum must be obtained for values

(r, ω) which lie outside the convex polytope determined by $\{(0,0), (\frac{2}{3}J_1^I, 0), (0, \frac{2}{3}J_2^I)\}$. Thus, this implies that there are no values of $(r, \omega, \varphi(r, \omega)) \in A_3$, with A_3 is the convex polytope determined by the set of angular points:

$$a_3 = \{(\frac{2}{3}J_1^I, 0, 0), (0, \frac{2}{3}J_2^I, 0), (0,0,0), (\frac{2}{3}J_1^I, 0, J_3^I), (0, \frac{2}{3}J_2^I, J_3^I), (0, 0, J_3^I)\},$$

which maximize the Nash-product.
This proof can be repeated for the case where J_2 is a function of J_1, J_3 which yields that there are no solutions possible inside A_2. In a similar way we get A_1. Thus the values $(J_1^{NB}, J_2^{NB}, J_3^{NB})$ which are determined by maximizing the Nash-product must lie inside cube C, but outside $\cup_{i=1}^3 A_i$.

To illustrate the truncated cube in the 3-player case we give in figure A.2 a representation

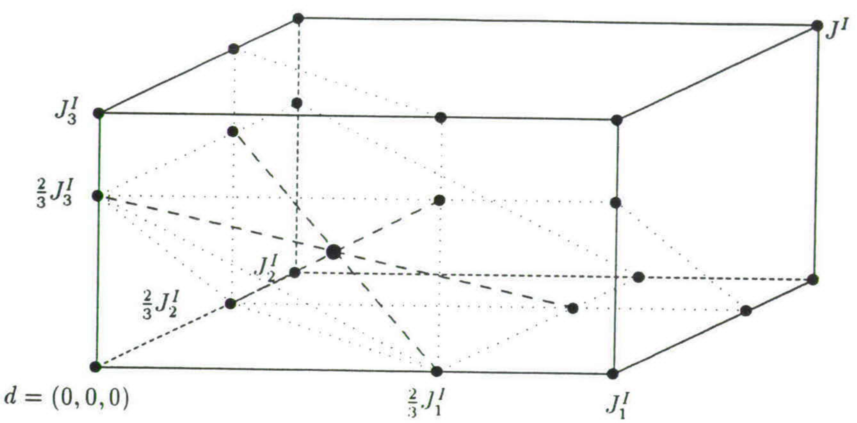


Figure A.2: A 3-dimensional representation of cube C, and the polytopes A_1, A_2 and A_3.

of this cube. The solid lines indicate cube C. The dotted lines inside the cube represent the convex polytopes A_1, A_2 and A_3. The three dashed lines inside the cube are the intersection lines of two of the three polytopes A_1, A_2 or A_3. Those three lines determine the point $\frac{1}{3}J^I$. The truncated cube $C \backslash \cup_{i=1}^3 A_i$ can now be determined by cutting the convex polytopes A_1, A_2 and A_3 from the cube C. This is done in figure A.3. In figure A.3 the truncated cube is determined by the solid- and dashed lines. The dotted lines represent the original cube. We see that the point $\frac{1}{3}J^I$ is a spearpoint of the truncated cube. Furthermore, we have that the truncated cube touches the polytope D, determined by the set $\{(J_1^I, 0, 0), (0, J_2^I, 0), (0, 0, J_3^I)\}$, in $\frac{1}{3}J^I$. Now, since the Pareto curve has to fall to the right

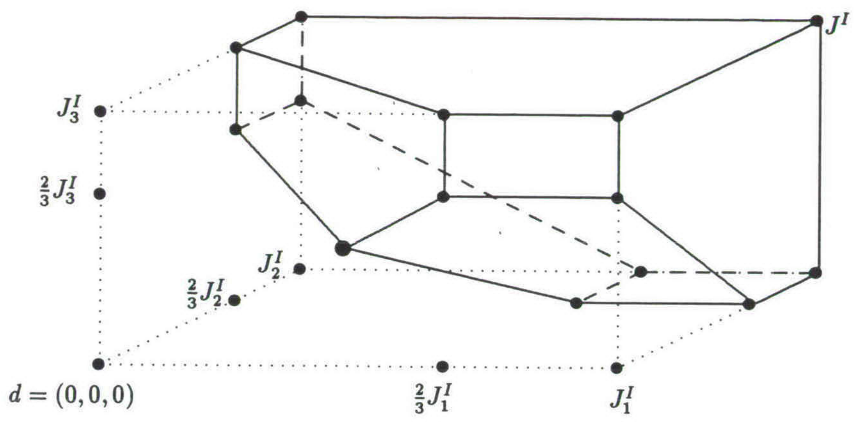


Figure A.3: The truncated cube in the 3-player case.

of this polytope D, we have that if the Pareto curve is relatively flat then the intersection of the Pareto curve and the truncated cube lies in the neighborhood of $\frac{1}{3}J^I$. Thus in that case the KS-solution and NB-solution will always lie 'fairly close'.

A.3.2 The N-player case

The proof for the N-player case is similar to the three player case. First, consider the Kalai-Smorodinsky solution. This solution is determined by the intersection between the line through the origin and the ideal point, λJ^I, and the convex polytope described by N angular points:

$$\{(J_1^I, 0, ..., 0), ..., (0, ..., 0, J_N^I)\}$$

Now, the intersection occurs for $\lambda = \frac{1}{N}$ and, again observe that the KS-solution can now be written as λJ^I with $\frac{1}{N} < \lambda \leq 1$. Observe, furthermore, that the KS-solution lies outside $A = \cup_{i=1}^N A_i$.

Secondly, consider the Nash bargaining solution. Write $J_N = \varphi(J_1, ..., J_{N-1})$. Now consider:

$$f(J_1, ..., J_{N-1}) = J_1 \cdots J_{N-1}\varphi(J_1, ..., J_{N-1}).$$

Transform the problem using spherical-coordinates. Let $\omega = (\omega_1, ..., \omega_{N-1})$. This yields:

$$J_i = r\omega_i,$$

for $i = 1, ..., N-1$. Define $prod{:=}\omega_1 \cdots \omega_{N-1}$, then we can rewrite f:

$$f(r,\omega) = r^{N-1}\varphi(r,\omega)prod.$$

Now, fix ω. Then

$$f'_r(r,\omega) = \{(N-1)r^{N-2}\varphi(r,\omega) + r^{N-1}\varphi'_r(r,\omega)\}prod.$$

Now, we are interested in points for which $f'_r = 0$, for $r > 0$. Again define g:

$$g(r) := (N-1)\varphi(r,\omega) + r\varphi'_r(r,\omega)$$

Observe now that $g(r)$ is monotone descending with $g(0) > 0$ and that $g(r_1) < 0$. Thus $g(r)$ has a maximum between $[0, r_1]$. Follow now the derivation of the proof in the 3-player case and remark that the maximum should be attained for $r \in [\frac{N-1}{N}r_1, r_1]$. After this observation the remaining part of the proof is straightforward.

A.4 Proof of Theorem 3.5.1

In this appendix we derive in the first subsection the strong d-monotonicity result for the KS-solution for the 3-player case. The argumentation of the proof for the N-player is the same. This will be done in the next subsection. In the third subsection we consider the NB-solution. We consider only the 3-player case and give a condition for which strong d-monotonicity holds in the 3-player case.

A.4.1 The KS-solution: 3-player case

First we consider the 3-player case for the KS-solution. Since, P can be represented by a strictly concave and differentiable function can write for every pair $(J_1, J_2, J_3) \in P$, $J_3 = \varphi(J_1, J_2)$. The KS-solution can now be determined by the equations:

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} + \lambda \begin{pmatrix} d_1 - J_1^I \\ d_2 - J_2^I \\ d_3 - J_3^I \end{pmatrix} = \begin{pmatrix} J_1^{KS} \\ J_2^{KS} \\ \varphi(J_1^{KS}, J_2^{KS}) \end{pmatrix}, \tag{A.6}$$

where the ideal point $J^I = (J_1^I, J_2^I, J_3^I)$ is determined by:

$$\begin{aligned} d_3 &= \varphi(J_1^I, d_2), \qquad \text{or} \qquad -d_3 + \varphi(J_1^I, d_2) = 0, \\ d_3 &= \varphi(d_1, J_2^I), \qquad \text{or} \qquad -d_3 + \varphi(d_1, J_2^I) = 0, \\ J_3^I &= \varphi(d_1, d_2). \end{aligned}$$

This implies that J_1^I and J_2^I are implicitly determined by a function of (d_1, d_2, d_3). Suppose now that

$$\begin{pmatrix} J_1^I \\ J_2^I \end{pmatrix} = \begin{pmatrix} \tilde{f}_1(d_1, d_2, d_3) \\ \tilde{f}_2(d_1, d_2, d_3) \end{pmatrix} = \tilde{f}(d_1, d_2, d_3),$$

then the implicit function theorem states that

$$\frac{\partial(J_1^I, J_2^I)}{\partial(d_1, d_2, d_3)} = \frac{\partial \tilde{f}}{\partial(d_1, d_2, d_3)} = \begin{pmatrix} 0 & -\frac{\varphi'_2}{\varphi'_1} & \frac{1}{\varphi'_1} \\ -\frac{\varphi'_1}{\varphi'_2} & 0 & \frac{1}{\varphi'_2} \end{pmatrix} \tag{A.7}$$

Remark that, here and in the sequel, we will use the notation φ'_i to denote the partial derivative of φ to the i'th component. From (A.6) follows now that:

$$d_3 + \lambda(d_3 - J_3^I) = \varphi(J_1^{KS}, J_2^{KS}) \qquad \text{or} \qquad \lambda = \frac{J_3^{KS} - d_3}{d_3 - J_3^I}.$$

Therefore, J_1^{KS}, J_2^{KS} are implicitly determined by:

$$\begin{cases} J_1^{KS} - d_1 - \frac{J_3^{KS} - d_3}{d_3 - J_3^I}(d_1 - J_1^I) = 0 \\ J_2^{KS} - d_2 - \frac{J_3^{KS} - d_3}{d_3 - J_3^I}(d_2 - J_2^I) = 0 \end{cases}$$

or, substituting

$$\begin{cases} g_1 = J_1^{KS} - d_1 + \frac{d_3 - \varphi(J_1^{KS}, J_2^{KS})}{d_3 - \varphi(d_1, d_2)}(d_1 - \tilde{f}_1(d_1, d_1, d_3)) = 0 \\ g_2 = J_2^{KS} - d_2 + \frac{d_3 - \varphi(J_1^{KS}, J_2^{KS})}{d_3 - \varphi(d_1, d_2)}(d_2 - \tilde{f}_2(d_1, d_1, d_3)) = 0 \end{cases} \tag{A.8}$$

Thus $g = (g_1, g_2)$ determines implicitly (J_1^{KS}, J_2^{KS}) as a function of (d_1, d_2, d_3). Now let

$$J_g = \left(\frac{\partial(g_1, g_2)}{\partial(J_1^{KS}, J_2^{KS})}, \frac{\partial(g_1, g_2)}{\partial(d_1, d_2, d_3)} \right).$$

We will now explicitly derive J_g, but in order to save space we, first, introduce the following notation:

$$I_i = \frac{d_i - J_i^I}{d_3 - J_3^I}, \qquad K_i = \frac{d_i - J_i^{KS}}{d_3 - J_3^I}$$

for $i = 1,2,3$. Then, computing the derivatives from (A.8) yields

$$\frac{\partial g}{\partial(J_1^{KS}, J_2^{KS})} = \begin{pmatrix} 1 - I_1\varphi'_1 & -I_1\varphi'_2 \\ -I_2\varphi'_1 & 1 - I_2\varphi'_2 \end{pmatrix}, \tag{A.9}$$

and

$$\frac{\partial g}{\partial(d_1, d_2, d_3)} = \begin{pmatrix} K_3 - 1 + \varphi'_1 I_1 K_3 & K_3\varphi'_2(I_1 + \frac{1}{\varphi'_1}) & I_1(1 - K_3) - \frac{1}{\varphi'_1}K_3 \\ K_3\varphi'_1(I_2 + \frac{1}{\varphi'_2}) & K_3 - 1 + \varphi'_2 I_2 K_3 & I_2(1 - K_3) - \frac{1}{\varphi'_2}K_3 \end{pmatrix}.$$

Now, since the matrix in (A.9) is always non-singular, the implicit function theorem states that

$$\frac{\partial(J_1^{KS}, J_2^{KS})}{\partial(d_1, d_2, d_3)} = -\{\frac{\partial g}{\partial(J_1^{KS}, J_2^{KS})}\}^{-1}\{\frac{\partial g}{\partial(d_1, d_2, d_3)}\} \tag{A.10}$$

where the inverse of the matrix in (A.9):

$$\{\frac{\partial g}{\partial(J_1^{KS}, J_2^{KS})}\}^{-1} = \frac{1}{1 - \{I_1\varphi'_1 + I_2\varphi'_2\}} \begin{pmatrix} 1 - I_2\varphi'_2 & I_1\varphi'_2 \\ I_2\varphi'_1 & 1 - I_1\varphi'_1 \end{pmatrix}.$$

Now we are able to derive explicitly the elements of the matrix in (A.10). Now, we first compute the upper-left element:

$$\frac{\partial J_1^{KS}}{\partial d_1} = -\frac{K_3 - 1 + 2I_1K_3\varphi'_1 + 2I_1I_2\varphi'_2 - I_2K_3\varphi'_2}{1 - \{I_1\varphi'_1 + I_2\varphi'_2\}}.$$

Now, remark that $K_3 - 1 < 0$ and $K_i < 0, I_i < 0$ for $i = 1,2,3$, and that, since φ is concave, $\varphi'_i < 0$, for $i = 1,2$. This implies that $\frac{\partial J_1^{KS}}{\partial d_1} > 0$. This result is in line with the result of Thomson, i.e., the Kalai Smorodinsky solution satisfies d-monotonicity. Now, after some extensive calculation we can derive:

$$\begin{aligned}
\frac{\partial J_1^{KS}}{\partial d_2} &= -\frac{\varphi'_2}{\varphi'_1} \cdot \frac{K_3 - I_1I_3\varphi'_1 + 2I_1K_3\varphi'_1 - I_2K_3\varphi'_2}{1 - \{I_1\varphi'_1 + I_2\varphi'_2\}}. \\
\frac{\partial J_1^{KS}}{\partial d_3} &= \frac{1}{\varphi'_1} \cdot \frac{K_3 - I_1I_3\varphi'_1 + 2I_1K_3\varphi'_1 - I_2K_3\varphi'_2}{1 - \{I_1\varphi'_1 + I_2\varphi'_2\}}.
\end{aligned}$$

Remark now, that since the sign of $-\frac{\varphi'_2}{\varphi'_1}$ and $\frac{1}{\varphi'_1}$ are both negative we have that the sign of both $\frac{\partial J_1^{KS}}{\partial d_2}$ and $\frac{\partial J_1^{KS}}{\partial d_3}$ must be the same. Since, the problem is symmetric in $\{J_1, J_2, J_3\}$ and symmetric in $\{d_1, d_2, d_3\}$ we have that the sign of every $\frac{\partial J_i^{KS}}{\partial d_j}$ for $i, j = 1,2,3, i \neq j$ must be the same. Now, observe that if this sign would be positive, each player would gain by a small positive perturbation of d_1; this is, due to the Pareto optimality condition, impossible. Thus, we can now construct the sign-matrix for the derivative:

$$\frac{\partial(J_1^{KS}, J_2^{KS}, J_3^{KS})}{\partial(d_1, d_2, d_3)} = \begin{pmatrix} + & - & - \\ - & + & - \\ - & - & + \end{pmatrix}.$$

This observation indicates that player 2 and 3 do not gain if we give a small positive perturbation to d_1, i.e., the Kalai Smorodinsky solution satisfies strong d-monotonicity.

A.4.2 The KS-solution: N-player case

The derivation of the proof of strong d-monotonicity in the N-player case is in its essence the same. First, write for every $J_1, ..., J_N \in P, J_N = \varphi(J_1, ..., J_{N-1})$. Follow now the previous proof, and remark that

$$\frac{(\partial J_1, ..., J_{N-1})}{\partial(d_1, ..., d_N)} = \begin{pmatrix} 0 & -\frac{\varphi'_2}{\varphi'_1} & \cdots & -\frac{\varphi'_{N-1}}{\varphi'_1} & \frac{1}{\varphi'_1} \\ -\frac{\varphi'_1}{\varphi'_2} & 0 & \cdots & -\frac{\varphi'_{N-1}}{\varphi'_2} & \frac{1}{\varphi'_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{\varphi'_1}{\varphi'_{N-1}} & \cdots & \cdots & 0 & \frac{1}{\varphi'_2} \end{pmatrix}$$

Now, observe that

$$\frac{\partial g}{\partial(J_1^{KS}, ..., J_{N-1}^{KS})} = I - \begin{pmatrix} I_1 \\ \vdots \\ I_{N-1} \end{pmatrix} \left(\varphi'_1, \cdots, \varphi'_{N-1}\right),$$

where I is the identity matrix. Due to this special form it is possible to calculate the inverse of this matrix explicitly:

$$\left\{\frac{\partial g}{\partial(J_1^{KS}, ..., J_{N-1}^{KS})}\right\}^{-1} = I + \frac{\begin{pmatrix} I_1 \\ \vdots \\ I_{N-1} \end{pmatrix} \left(\varphi'_1, \cdots, \varphi'_{N-1}\right),}{1 - (\varphi'_1, \cdots, \varphi'_{N-1}) \begin{pmatrix} I_1 \\ \vdots \\ I_{N-1} \end{pmatrix}} \tag{A.11}$$

After some extensive calculation it is possible to derive $\frac{\partial g}{\partial d}$. For the proof, however, we are just interested in the second and third column of this matrix. These are given by

$$\frac{\partial g}{\partial(d_2,d_3)} = \begin{pmatrix} K_N{\varphi'}_2(I_1+\frac{1}{{\varphi'}_1}) & K_N{\varphi'}_3(I_1+\frac{1}{{\varphi'}_1}) \\ K_N - I_N + {\varphi'}_2 I_2 K_N & K_N{\varphi'}_3(I_2+\frac{1}{{\varphi'}_2}) \\ K_N{\varphi'}_2(I_3+\frac{1}{{\varphi'}_3}) & K_N - I_N + {\varphi'}_3 I_3 K_N \\ \vdots & \vdots \\ K_N{\varphi'}_2(I_{N-1}+\frac{1}{{\varphi'}_{N-1}}) & K_N{\varphi'}_3(I_{N-1}+\frac{1}{{\varphi'}_{N-1}}) \end{pmatrix}$$

We can calculate and compare $\frac{\partial J_1^{KS}}{\partial d_2}$ and $\frac{\partial J_1^{KS}}{\partial d_3}$. Remark, that we only need the first row of the matrix in (A.11) for deriving these expressions. This yields that

$${\varphi'}_3\frac{\partial J_1^{KS}}{\partial d_2} = {\varphi'}_2\frac{\partial J_1^{KS}}{\partial d_3}.$$

This observation implies that signs of both terms, $\frac{\partial J_1^{KS}}{\partial d_2}$ and $\frac{\partial J_1^{KS}}{\partial d_3}$, are the same. Now, we use the symmetry argument to derive that all terms $\frac{\partial J_i^{KS}}{\partial d_j}, j \neq i, i, j = 1, ..., N$ must have the same sign. Since we are looking after Pareto optimal outcomes, it is impossible that the signs are all positive; thus we, finally, have that

$$\frac{\partial J_i^{KS}}{\partial d_i} > 0, \qquad \text{and} \qquad \frac{\partial J_i^{KS}}{\partial d_j} < 0,$$

for $i = 1, ..., N$, and $j \neq i$, which yields that the Kalai Smorodinsky satisfies strong d-monotonicity in the N-player case.

A.4.3 The Nash bargaining solution

Consider the 3-player case. Since P is concave, there is a function φ such that $(J_1, J_2, J_3) = (J_1, J_2, \varphi(J_1, J_2)) \in P$. The Nash bargaining solution is determined by

$$\max_{J_1,J_2}(J_1 - d_1)(J_2 - d_2)(\varphi(J_1, J_2) - d_3)$$

This maximization problem contains, according to Nash, exactly one global maximum. Furthermore, it is clear that the solution of this problem, say $J^{NB} = (J_1^{NB}, J_2^{NB}, J_3^{NB})$, lies not on the edge of P, i.e., it is an internal element of P. Thus the Nash bargaining solution is uniquely determined by:

$$\begin{aligned} (J_2^{NB} - d_2)\{\varphi(J_1^{NB}, J_2^{NB}) - d_3 + {\varphi'}_1(J_1^{NB} - d_1)\} &= 0 \\ (J_1^{NB} - d_1)\{\varphi(J_1^{NB}, J_2^{NB}) - d_3 + {\varphi'}_2(J_2^{NB} - d_2)\} &= 0. \end{aligned}$$

Now, we follow the same procedure as in the proof of the Kalai Smorodinsky solution. This yields that there is a function g for which $g_i(J_1^{NB}, J_2^{NB}, d_1, d_2, d_3) = 0$ for $i = 1, 2$, with

$$\begin{cases} g_1 = \varphi(J_1^{NB}, J_2^{NB}) - d_3 + \varphi'_1(J_1^{NB} - d_1) = 0 \\ g_2 = \varphi(J_1^{NB}, J_2^{NB}) - d_3 + \varphi'_2(J_2^{NB} - d_2) = 0 \end{cases}$$

Thus

$$\frac{\partial g}{\partial(J_1^{NB}, J_2^{NB})} = \begin{pmatrix} 2\varphi'_1 + \varphi''_{11}(J_1^{NB} - d_1) & \varphi'_2 + \varphi''_{12}(J_1^{NB} - d_1) \\ \varphi'_1 + \varphi''_{21}(J_2^{NB} - d_2) & 2\varphi'_2 + \varphi''_{22}(J_2^{NB} - d_2) \end{pmatrix}, \tag{A.12}$$

and

$$\frac{\partial g}{\partial(d_1, d_2, d_3)} = \begin{pmatrix} -\varphi'_1 & 0 & -1 \\ 0 & -\varphi'_2 & -1 \end{pmatrix}.$$

Now, suppose that $\frac{\partial g}{\partial(J_1^{NB}, J_2^{NB})}$ is invertible, then its inverse is given by

$$\{\frac{\partial g}{\partial(J_1^{NB}, J_2^{NB})}\}^{-1} = \frac{1}{det} \begin{pmatrix} 2\varphi'_2 + \varphi''_{22}(J_2^{NB} - d_2) & -\varphi'_2 + \varphi''_{12}(J_1^{NB} - d_1) \\ -\varphi'_1 + \varphi''_{21}(J_2^{NB} - d_2) & 2\varphi'_1 + \varphi''_{11}(J_1^{NB} - d_1) \end{pmatrix},$$

where det is the determinant of the matrix in (A.12). Now, we are ready to calculate the behaviour of the Nash bargaining solution if we perturbate (d_1, d_2, d_3). This is determined by

$$\frac{\partial(J_1^{NB}, J_2^{NB})}{\partial(d_1, d_2, d_3)} = -\{\frac{\partial g}{\partial(J_1^{NB}, J_2^{NB})}\}^{-1}\{\frac{\partial g}{\partial(d_1, d_2, d_3)}\} \tag{A.13}$$

Now, observe that $\frac{\partial J_1^{NB}}{\partial d_1} > 0$ and $\frac{\partial J_2^{NB}}{\partial d_2} > 0$ which is in line with the d-monotonicity result of Thomson [78]. However, observe also if

$$-\varphi'_2 + \varphi''_{12}(J_1^{NB} - d_1) > 0$$

then $\frac{\partial J_1^{NB}}{\partial d_2} > 0$. This indicates that player two gains if we give small positive perturbation to d_2. From this result we can derive, for the 3-player case, a necessary condition for strong d-monotonicity which is that $\varphi''_{ij} > 0$. Furthermore, remark that for the general N-player case, the derivation of (A.13) is much more complicated, since this involves computing the inverse of $\frac{\partial g}{\partial(J_1^{NB}, \dots, J_{N-1}^{NB})}$.

Appendix B

Appendices of chapter 4

B.1 Description of data and data source

Our data source contains yearly data from 1960 till 1991. Most of the data are taken from the OECD: OECD Economic Outlook 53, Statistics on microcomputer diskette nr. 53, with the exception of government expenditure and real taxes (or receipts government), which we took from the European Economy 51 (EE 51), May 1992. The data for the short term interest rate are taken from the IFS 92 (International Financial Statistics 1992). The short term interest rate data are not very reliable for the period 1960-1970 where we sometimes had to rely on the discount rates. Below we will give an exact description of the data for each variable separately, and, subsequently, we will give for each country separately the way how we constructed the missing data.

Y : Gross national/domestic product, volume (OECD: GDPV)
G : Total expenditure General Government (EE 51, given as percentage of Y)
T : Current receipts General government (EE 51, given as percentage of Y)
RS : Nominal short term interest rate (OECD: IRS, if missing: IFS 92)
RL : Nominal long term interest rate (OECD: IRL)
E : Exchange rate (OECD: EXCH)
P_y : Deflator for GDP at market prices (OECD: PGDP)
P_c : Deflator for consumer expenditure (OECD: PCP)
W : Wage compensation per employee, private sector (OECD: WSSE)
L : Labour force (OECD: LF)
N : Total employment (OECD: ET)
U : Unemployment rate (OECD: UNR)

Most data are available, however there were some specific problems for W and RS. For

W we followed the approach of Heylen [41]. If W was missing, we used as representative growth rate for W the growth rate of compensation per employee in the total economy which are listed in the European Economy. The assumption made is that during that period the growth rate of both variables is identical. For the RS we relied on IFS data, where we took the discount rate or money market rate. Finally, the trend output was constructed from the gross GDP variable, trend output was calculated with the following regression: $Y = \alpha_0 + \alpha_1 \text{time} + \alpha_2$ DUM7475$+\epsilon_t$, where DUM7475 is one during the years 1960- 1973 and zero during the years 1974-1991 (see, e.g., Perron [64]). The explanation for the dummy DUM65 in the employment equation in Italy is explained by Heylen [41]: "Dummy for the extensive government program to fight the recession of 1963-64 (OCDE, April 1966, pp. 11-14.)." Heylen [41] also explains the dummy DUM70 in the wage equation of Germany: 'Dummy variable captures the effects of the deteriotion of the social climate (e.g. wildcat strikes in the autumn of 1969) and growing union militancy (to reverse the trend of declining labour shares) (OECD , Perspectives Economiques, Paris, OCDE, June 1971, pp. 13-14).'

Country specific remarks:

Belgium: Data on W were only available since 1970. For the 1960s we used the approach as given above. The exchange rate E was also only available since 1970, for the 1960s we used IFS data (market rate, wf).

France: Data on W for 1960-1962 is based on the European Economy and for RS, from 1960-1969, we used IFS data (money market rate, 60b).

Denmark: OECD Data for RS was only available from 1979, so before that period we used IFS data (discount rate, 60).

Germany: All data, as indicated above, available.

United Kingdom: Data on W was missing for the period 1960-1961; for these two years we used the the approach as stated above. For the RS we used from 1960-1969 IFS data (Eurodollar rate, 60d).

Ireland: IFS data for the RS was used from 1960-1969 (discount rate, 60).

Italy: For the RS we used from 1960-1968 the discount rate (60) and for the period 1969-1970 the money market rate (60b) of the IFS data.

Netherlands: For the period 1960-1969, W was calculated as stated above.

USA: All data as given above available.

Japan: For the period 1960-1964, W was calculated as stated above. For 1960-1968, RS was taken from the IFS data (money market rate, 60b) and for 1960-1962, we used IFS data (lending rate, 60p) for RL. Not available were government expenditure and taxes for the years 1960-69. These where approximated with calculated growth rates. These growth rates were calculated with the use of OECD data, where government expenditure is calculated from current disbursements of government (YPG) and taxes from current receipts of government (YRG).

Appendix C

Appendices of chapter 5

C.1 Derivation of cooperative convergence solutions

In this appendix we derive the formula for computing cooperative convergence outcomes (CCO). To that end the convergence problem is rewritten into a standard optimal control problem. First, however, we will formulate the description of the dynamic behaviour of each country in the SLIM-model:

Assumption C.1 *The economic behaviour of the individual countries can be described by (for $i = 1, \ldots, N$) :*

$$y_i(t) = A_i y_i(t-1) + \sum_{j \neq i}^{N} A_{ij} y_j(t-1) + B_i u_i(t) + D_i d_i(t) \tag{C.1}$$

where $y_i(t) \in \mathbb{R}^{n_i}$ is the state of the i-th country (endogenous variables), $u_i(t) \in \mathbb{R}^{m_i}$ is the control vector (instrumental variables) and the vector $d_i(t) \in \mathbb{R}^{\ell_i}$ is the purely exogenous data-vector. For all i, A_i, A_{ij}, B_i and D_i are real matrices of appropriate dimensions.

Assumption C.2 *Every country solves the problem:*

$$\min_{u_i} J_i := \min_{u_i} \sum_{t=t_0}^{t_f} \left\{ \|y_i(t) - y_i^*(t)\|^2_{Q_i(t)} + \|u_i(t) - u_i^*(t)\|^2_{R_i(t)} \right\}$$

subject to (C.1).

Using the stacked forms $u(t) = (u_1'(t), ..., u_N'(t))'$ and $y(t) = (y_1'(t), ..., y_N'(t))'$ the aspect of convergence can be described as follows:

Assumption C.3 *The minimization problem with respect to the convergence function is defined as:*

$$\min_u C(u) := \min_u \sum_{t=t_0}^{t_f} \|L(t)y(t)\|^2_{Q_0(t)}$$

Remark. The matrices $L(t), Q_0(t)$ can be chosen dependent on the problem. For instance if we want to investigate convergence to the average of the endogenous variables, the matrices L can be specified as follows (for $t = t_0, ..., t_f$):

$$L = \begin{pmatrix} \frac{N-1}{N}I & -\frac{1}{N}I & . & . & . & -\frac{1}{N}I \\ -\frac{1}{N}I & \frac{N-1}{N}I & -\frac{1}{N}I & & & -\frac{1}{N}I \\ . & & . & & & . \\ . & & & . & & . \\ . & & & & . & . \\ -\frac{1}{N}I & . & . & . & -\frac{1}{N}I & \frac{N-1}{N}I \end{pmatrix}$$

or if we want to investigate convergence to endogenous variables of a specific country, say country i, we use

$$L = \begin{pmatrix} -I & 0 & \ldots & 0 & I & 0 & \ldots & 0 \\ 0 & -I & 0 & \ldots & I & 0 & \ldots & 0 \\ . & & . & & . & & & . \\ . & & & . & I & & & . \\ 0 & . & . & 0 & 0 & 0 & \ldots & 0 \\ . & & & & I & . & & . \\ . & & & & . & & . & . \\ 0 & . & . & . & I & . & . & -I \end{pmatrix}$$

The matrices $Q_0(t)$ give the weights that each country wants to assign to the convergence function in each time period. Remark, that in the paper we choose L such that it satisfies the convergence function as presented in the paper.

Assumption C.4 (cooperative convergence problem) *Given*

$$0 \leq \alpha_i \leq 1, \quad i = 1, ..., N, \quad \sum_{i=1}^{N} \alpha_i = 1, \quad 0 \leq \lambda \leq 1,$$

the problem to be solved is:

$$\min_{u_i} \quad (1-\lambda)\sum_{i=1}^{N}\alpha_i J_i \; + \; \lambda C$$

subject to (C.1) *for (i=1,...,N).*

The solution of the above stated control problem can be derived by reformulating it as a standard LQ problem. We first introduce the overall system vectors:

$$\begin{aligned}
y(t) &= (y_1'(t), y_2'(t), ..., y_N'(t))' \\
y^*(t) &= (y_1'^*(t), y_2'^*(t), ..., y_N'^*(t))' \\
u(t) &= (u_1'(t), u_2'(t), ..., u_N'(t))' \\
u^*(t) &= (u_1'^*(t), u_2'^*(t), ..., u_N'^*(t))' \\
d(t) &= (d_1'(t), d_2'(t), ..., d_N'(t))'
\end{aligned}$$

Next introduce matrices $Q(t), R(t), Q^*(t), A, B$ and D in the following way:

$$\begin{aligned}
Q(t) &= (1-\lambda)\mathrm{diag}(\alpha_1 Q_1(t), \alpha_2 Q_2(t), ..., \alpha_N Q_N(t)) \\
R(t) &= (1-\lambda)\mathrm{diag}(\alpha_1 R_1(t), \alpha_2 R_2(t), ..., \alpha_N R_N(t)) \\
Q^*(t) &= \lambda L'(t) Q_0(t) L(t) \\
A &= \begin{pmatrix} A_1 & A_{12} & . & . & . & A_{1N} \\ A_{21} & A_2 & & & & . \\ . & & & . & & . \\ . & & & & . & . \\ . & & & & . & . \\ A_{N1} & . & . & . & . & A_{NN} \end{pmatrix} \\
B &= \begin{pmatrix} B_1 & 0 & . & . & . & 0 \\ 0 & B_2 & & & & . \\ . & & . & & & . \\ . & & & . & & . \\ . & & & & . & 0 \\ 0 & . & . & . & 0 & B_N \end{pmatrix} \\
D &= \begin{pmatrix} D_1 & 0 & . & . & . & 0 \\ 0 & D_2 & & & & . \\ . & & . & & & . \\ . & & & . & & . \\ . & & & & . & 0 \\ 0 & . & . & . & 0 & D_N \end{pmatrix}
\end{aligned}$$

The above problem is equivalent with the optimal control problem:

$$\min_{\bar{u}} \sum_{t=t_0}^{t_f} \left\{ \|\bar{y}(t) - \bar{y}^*(t)\|^2_{\bar{Q}(t)} + \|\bar{u}(t) - \bar{u}^*(t)\|^2_{\bar{R}(t)} \right\}$$

subject to

$$\bar{y}(t) = \bar{A}\bar{y}(t-1) + \bar{B}\bar{u}(t) + \bar{D}\bar{d}(t),$$

with

$$\begin{aligned}
\bar{A} &= \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} \\
\bar{B} &= \begin{pmatrix} B \\ B \end{pmatrix} \\
\bar{D} &= \begin{pmatrix} D \\ D \end{pmatrix} \\
\bar{Q}(t) &= \begin{pmatrix} Q(t) & 0 \\ 0 & Q^*(t) \end{pmatrix} \\
\bar{R}(t) &= R(t) \\
\bar{y}(t) &= \begin{pmatrix} y(t) \\ y(t) \end{pmatrix} \\
\bar{u}(t) &= \begin{pmatrix} u(t) \end{pmatrix} \\
\bar{y}^*(t) &= \begin{pmatrix} y^*(t) \\ 0 \end{pmatrix} \\
\bar{u}^*(t) &= \begin{pmatrix} u^*(t) \end{pmatrix} \\
\bar{d}(t) &= \begin{pmatrix} d(t) \end{pmatrix}
\end{aligned}$$

Theorem C.5 *The solution for the cooperative convergence problem is than given, for $t = t_0, \ldots, t_f$, by :*

$$\hat{u}(t) = E(t+1)^{-1}\left(\bar{R}(t)\bar{u}^*(t) - \frac{1}{2}\bar{B}^T K(t+1)\left[\bar{A}\bar{y}(t-1) + \bar{D}\bar{d}(t)\right] - \frac{1}{2}\bar{B}^T g(t+1)\right)$$

where $K(t)$ satisfies, for $t = t_0, \ldots, t_f + 1$, the following backward Riccati difference equation :

$$\begin{cases} K(t) &= 2\bar{Q}(t-1) + \bar{A}^T K(t+1)\left(I - \frac{1}{2}\bar{B}E^{-1}(t+1)\bar{B}^T K(t+1)\right)\bar{A} \\ K(t_f+1) &= 2\bar{Q}(t_f) \end{cases}$$

$E(t+1)$ is defined, for $t = t_0, \ldots, t_f$, by :

$$E(t+1) := \bar{R}(t) + \frac{1}{2}\bar{B}^T K(t+1)\bar{B}$$

$g(t)$ satisfies, for $t = t_0, \ldots, t_f + 1$, the following backward difference equation :

$$\begin{cases} g(t) & = & -2\bar{Q}(t-1)y^*(t-1) + \bar{A}^T K(t+1)\bar{B}E^{-1}(t+1)\bar{R}(t)\bar{u}^*(t) \\ & & +\bar{A}^T K(t+1)\bar{D}d(t) - \frac{1}{2}\bar{A}K(t+1)\bar{B}E^{-1}(t+1)\bar{B}^T K(t+1)\bar{D}d(t) \\ & & +\bar{A}^T g(t+1) - \frac{1}{2}\bar{A}K(t+1)\bar{B}E^{-1}(t+1)\bar{B}^T g(t+1) \\ g(t_f+1) & = & -2\bar{Q}(t_f)\bar{y}^*(t_f) \end{cases}$$

Remark, that the standard cooperative problem, without convergence, can be computed by substituting $L = 0$ (see also Engwerda [27] and de Zeeuw [85]).

The CCO solution is now represented by a particular choice of the weights $\alpha^{CCO} = (\alpha_1, ..., \alpha_N, \lambda)$. To find this set we have to use a constraint optimization procedure. These procedures are available in existing computer packages. Since, in chapter two is proven that the CCO outcome coincides with the NC outcome in the $J_1, ..., J_N$-plane, the stopping criterium of the numerical optimization algorithm can be implemented as follows. Stop, if for all $i, i = 1, ..., N$, J_i is 'close' to J_i^{NC}.

C.2 Description of exogenous values

In this appendix we describe our choices for the exogenous values. For the two foreign countries, USA and Japan, we used as starting values, the true 1991 values. From thereon we constructed the exogenous values for 1992-1999, using linear interpolation. Since, links between countries in the SLIM-model are of three types: first, financial variables such as interest rates and exchange rates; second, GDP inflation; and third foreign output, we present in table C.1 just the (growth) rates for these values. Remark, that we assumed that the nominal long term interest rate, RL, is constant for the planning period 1992-1999. For

Table C.1: The exogenous (growth) rates for USA and Japan in 1999.

	rates for endogenous values		
Countries	ΔY	ΔP_y	RL
USA	2.00	3.00	8.00
Japan	3.00	2.00	6.00

the other exogenous variables in the model, the nominal exchange rate and the labour force we constructed the exogenous paths as follows. We used the actual 1991 values and from thereon we assumed for the labour force the average historical growth rates over the last ten years 1982-1991 and for the nominal exchange rates we assumed zero growth rates.

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Samenvatting

De invloed van de Europese Commissie is op veel gebieden zichtbaar. Zo ziet men regelmatig de deelnemende Europese landen bij elkaar komen voor het maken van onderlinge afspraken omtrent nieuwe wetten en maatregelen op zowel nationaal als internationaal terrein. Een van die bijeenkomsten vond plaats in Maastricht 1991. Tijdens deze bijeenkomst werden afspraken gemaakt tussen de deelnemende landen opdat men in 1999 gezamenlijk kan overstappen naar één munt. De afspraken tijdens dat verdrag, die in dit proefschrift onder de loep worden genomen, worden ook wel de convergentie condities genoemd. De condities waaraan ieder land moet voldoen om in 1999 toe te kunnen treden tot een Europese en Monetaire Unie kunnen als volgt worden samengevat:
(i) de consumptie prijs inflatie mag niet meer dan 2% boven het gemiddelde van de drie landen met de laagste consumptie prijs inflatie uitstijgen.
(ii) de nominale lange rente mag niet meer dan 2% boven het gemiddelde van de drie landen met de laagste consumptie prijs inflatie uitstijgen.
(iii) geen aanpassingen in de wisselkoersen voor tenminste twee jaar.
(iv) het overheidstekort en de overheidsschuld mogen niet te hoog zijn.
De methodologie die in dit proefschrift gebruikt wordt om deze convergentie eisen te analyseren is speltheorie. We veronderstellen dat de deelnemende Europese landen een spel met elkaar spelen waarbij ieder land zijn eigen doelstellingen probeert te maximaliseren. In het tweede hoofdstuk veronderstellen we dat de landen volledige informatie omtrent elkaars doelstellingen en structuur hebben en samenwerken om hun eigen doelen te bereiken. Een ideale coöperatieve spelsituatie dus. In zulk een omgeving lijkt de invloed van een Europese Commissie niet gewenst daar elke inmenging van zulk een coördinator het optimale evenwicht wel eens zou kunnen verstoren. Als we echter veronderstellen dat de Europese Commissie de sleutel tot optimale samenwerking bezit dan is de situatie interessant om een idee te krijgen in hoeverre de Europese Commissie haar convergentie doelstellingen kan opleggen aan de deelnemende landen. Het moge duidelijk zijn dat, als de Europese Commissie haar eisen te streng doorvoert, sommige landen te ver moeten afwijken van hun doelstellingen hetgeen zou kunnen resulteren in een gedrag waarbij deze landen liever de eigen doelstellingen volgen (en dus geen rekening wensen te houden met de convergentie

eisen). Het evenwicht dat in dit hoofdstuk geconstrueerd wordt is het evenwicht waarbij de convergentie maximaal is en waarbij ieder afzonderlijk land beter af is dan wanneer ieder land een onafhankelijk beleid zou volgen. Men zou kunnen zeggen dat dit evenwicht de maximaal haalbare convergentie aangeeft die tussen de landen mogelijk is. Ofwel, voldoen de landen in dit evenwicht niet aan de convergentie eisen zoals opgesteld in het verdrag van Maastricht dan is het bestaan van een EMU in 1999 met alle van de nu deelnemende landen onwaarschijnlijk.

Het moge duidelijk zijn dat de praktijk vele malen gecompliceerder is dan hierboven geschetst. Het is interessant om te onderzoeken hoe dit evenwicht zich gedraagt als er bijvoorbeeld andere informatie structuren of andere gedragsregels verondersteld worden tussen de landen. Zo wordt wel verondersteld dat de Europese landen nou niet bepaald een coöperatief spel spelen, maar dat de Europese Commissie juist in het leven is geroepen om landen tot meer coöperatief gedrag aan te sporen. Stel nu eens dat we uitgaan van een situatie waar de Europese landen niet coöperatief gedrag vertonen, dan kunnen we nagaan of het opleggen van de convergentie eisen de landen dichter bij een coöperatief evenwicht brengt. Dit is het geval als de negatieve externe effecten die er tussen de landen bestaan verzwakt en/of de positieve externe effecten tussen de landen versterkt worden.

In hoofdstuk drie worden eigenschappen van bepaalde coöperatieve evenwichten bij een spel van twee of meer spelers bestudeerd. Indien men in de praktijk coöperatieve evenwichten wil uitrekenen dan wordt vaak het onderhandelins-evenwicht van Nash of van Kalai-Smorodinsky genomen. In dit hoofdstuk wordt allereerst een algoritme ontworpen om de Nash onderhandelings-uitkomst snel uit te rekenen in het geval van meer dan twee spelers. Verder wordt nagegaan hoe groot de verschillen tussen de twee bovengenoemde onderhandelings-uitkomsten in de praktijk zijn en in het theoretische geval wordt er een gebied aangegeven waarin beide uitkomsten altijd moeten liggen. Verder wordt er nagegaan wat het gedrag van de twee uitkomsten is bij kleine veranderingen van het dreigpunt.

Om deze ideeën na te gaan in de praktijk is er in hoofdstuk vier een klein meerlanden model, genaamd SLIM, ontwikkeld dat acht landen van de Europese Unie, alsmede de USA en Japan beschrijft. Het model bevat zes gedragsvergelijkingen per land en is geschat met behulp van jaarlijkse data van 1960-1991. De structuur die opgelegd is aan ieder land is afkomstig van de economen Mundell en Fleming. Een bijzondere eigenschap van het model is dat het in vergelijking met andere meerlandenmodellen sterkere interactie toestaat tussen landen.

Het uitrekenen van speltheoretische evenwichten komt aan de orde in hoofdstuk vijf. In dit hoofdstuk wordt vooral aandacht besteed aan de twee centrale convergentie criteria: convergentie in consumptie inflatie prijzen en convergentie in nominale lange rentes. Uitgaande van een coöperatieve spelsituatie blijkt er heel wat convergentie tussen de landen mogelijk te zijn. Het model suggereert in een bepaalde uitkomst echter dat Duitsland, Nederland en België een iets hoger nominaal rente beleid en wat meer inflatie in de consumptie prijzen moeten toelaten om zodoende de andere EU-landen (Denemarken, Frankrijk, Ierland, Italië, UK in het SLIM-model) meer ruimte te verschaffen om ook werkelijke convergentie te kunnen bewerkstelligen. Hiermee wordt dus tevens gesuggereerd dat als, bijvoorbeeld, Duitsland toch een strikt monetair beleid met lage inflatie wenst te volgen dat convergentie voor de andere EU-landen moeilijker wordt en dat voor die EU-landen het eigen beleid in het gedrang zou kunnen komen. Tevens zouden dan de kosten voor fase twee van de EMU, om daadwerkelijk over te gaan op één munt, ook hoger zijn voor de EU als geheel. Het opleggen van de convergentie eisen in een niet coöperatief spel heeft als resultaat dat sommige landen beter en andere landen slechter af zijn dan in het geval van een niet coöperatief spel zonder convergentie eisen. Er is zo bezien dus geen aanwijzing dat het opleggen van de convergentie eisen negatieve dan wel positieve externe effecten versterkt dan wel afzwakt. Het blijkt zelfs zo te zijn dat het opleggen van de convergentie eisen in een niet coöperatief spel, convergentie nauwelijks doet toenemen. Met andere woorden convergentie is alleen mogelijk als er tenminste een bepaalde graad van samenwerking tussen de landen is. Een andere uitkomst van het experiment is dat de landen Duitsland en Frankrijk het grootste voordeel hebben bij samenwerking. Het experiment suggereert dat, aangezien deze twee landen de meeste invloed hebben in de Europese Unie, voor hun ook het verschil in welzijn tussen een coöperatieve en niet coöperatieve oplossing het grootst is. Opmerkelijk zijn de geringe voordelen van het Verenigd Koninkrijk bij samenwerken (t.o.v. niet samenwerken), hetgeen suggereert dat het Verenigd Koninkrijk niet erg geïnteresseerd is in samenwerking met de andere EU-landen.

De resultaten in dit proefschrift moeten gezien worden als een eerste aanzet tot verdere analyse van deze interessante problematiek. Het onderliggende proefschrift geeft, in beknopte zin, weer hoe men de convergentie-problematiek zou kunnen analyseren in de theorie en in de praktijk.

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Eindhoven. He graduated in the field of experimental design. His PhD research at Tilburg University, for the Netherlands Organization for Scientific Research (NWO) covered the fields of econometrics, game theory and macroeconomics. Rudy Douven will be taking up a position at the ESRC Macroeconomic Modelling Bureau at Warwick University.

This thesis addresses theoretical and empirical aspects of the impact of the Maastricht Treaty's (1991) convergence conditions on the EU-Member States. The convergence conditions are modelled as a restriction on a dynamic game between the EU-Member States. The theoretical part analyses cooperative games. In the empirical part a small linear interdependent multi-country model of eight EU-Member States, USA and Japan is developed. Some of the empirical results suggest that coordination is more profitable for the larger (leading, and more interdependent) economies, such as Germany and France, than for the smaller (following, and more dependent) economies, such as Belgium, Denmark and the Netherlands.

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